Argument Theory Change: Revision Upon Warrant

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Abstract. We propose an abstract argumentation theory whose dynamics is captured by the application of belief revision concepts. The theory is deemed as abstract because both the underlying logic for arguments and argumentative semantics remain unspecified. Regarding our approach to argument theory change, we define some basic change operations along with their necessary theoretical elements towards the definition of a warrant-prioritized revision operation. This kind of revision expands the theory by an argument and then applies a contraction ensuring that the added argument can be believed afterwards.

Keywords. Formalization of abstract argumentation, Applications

1. Introduction & Motivations

In this article, we introduce an abstract theory that captures the dynamics of a proposed argumentation framework through the application of belief revision concepts. Often in the literature, abstract argumentation frameworks [1,2] are built on top of Dung's [3], which does not consider dynamics. Therefore, we define a dynamic abstract argumentation theory including dialectical constraints, and then we present argument revision techniques to describe the fluctuation of the set of active arguments (the ones considered by the inference process of the theory). We claim that our theory is abstract from two standpoints: (1) there is no restriction to any particular representation for arguments nor argumentative semantics, (2) we provide a characterization of the change operators (specially contractions), which is not restricted to a particular implementation.

Belief revision has been applied to an argumentation system in a previous work [4], but in a rather different direction than the proposed here. In [4], a non-prioritized revision is applied in order to insert (partially or totally) a given explanation (a minimal proof) for a sentence to the knowledge base. If the explanation is partially accepted, it is then recognized as an argument (a "defeasible" proof). A closer approach to the one here presented was given in [5], where a non-abstract preliminary investigation on the argument revision matter was introduced.

In our approach, we define expansion, contraction, and revision operators, where the latter can be expressed in terms of the other two, leading to an identity similar to the one defined by Isaac Levi [6]. Our goal is to define an abstract theory that allows for the introduction of an argument ensuring it can be believed afterwards. This is achieved by applying a revision, that is, an expansion followed by a contraction. The expansion operator is quite straightforward, but (as usual in any model for the theory change) the main complexity relies on the definition of the contraction operator, which allows a wide range of possibilities: from affecting unrestrictedly any number of arguments in the system to keeping this perturbation to a minimum. This choice is up to the minimal change principle followed by the specification of the contraction operation, which also has an indirect impact over the attack relation among arguments.

This paper is organized as follows: first, we introduce the dynamic abstract argumentation framework, then the argument theory change is defined, giving a very brief overview of classic belief revision, and finally the argument change operators are defined, along with an inter-definition between revision and contraction/expansion.

2. A Dynamic Abstract Framework with Dialectical Constraints

The framework proposed in this work is formed by arguments, possible ways of interrelating them, and a way of identifying those that are going to be part of the argumentative process. After presenting the dynamic abstract framework, we will give the specific notion of argument used in this article.

Definition 1 (Dynamic Argumentation Framework) *A **dynamic argumentation framework** (DAF) Φ is a tuple $\langle \mathbb{U}, \mathbb{A}, \mathbf{R}, \sqsubseteq \rangle$, where $\mathbb{U}$ is a finite set of arguments called **universal**, $\mathbb{A} \subseteq \mathbb{U}$ is called the **set of active arguments**, $\mathbf{R} \subseteq \mathbb{U} \times \mathbb{U}$ denotes an **attack relation** between arguments, and $\sqsubseteq$ is a partial order over $\mathbb{U}$ called the **subargument relation**.*

The universal set of arguments $\mathbb{U}$ characterizes the full set of arguments that could appear in a given domain. At a given instant, the set $\mathbb{A}$ of active arguments will represent the complete pool of arguments that can be used by the system to make inferences. Note that $\mathbb{U}$, $\mathbf{R}$, and $\sqsubseteq$ are deemed as static, whereas the content of $\mathbb{A}$ is dynamic, since any change in the system (*i.e.*, its dynamics) is reflected into this set. Having both the universal set of arguments and the subset of the currently active ones allows us to identify the *subset of inactive arguments*, *i.e.*, $\mathbb{I} = \mathbb{U} \setminus \mathbb{A}$. The set of inactive arguments will contain the remainder of arguments (in the universal set) that is not considered by the argumentative process at a specific instant. Later in this article, we will show how inactive arguments can be activated.

In the rest of the article, to refer to attacks between arguments, we will simply use the word "attacks" or the notation $\mathcal{A}_1\mathbf{R}\mathcal{A}_2$, which means that $\mathcal{A}_1$ attacks $\mathcal{A}_2$, or equivalently that $\mathcal{A}_1$ is a defeater for $\mathcal{A}_2$. As in [7], the symbol $\sqsubseteq$ denotes subargument relation: $\mathcal{A} \sqsubseteq \mathcal{B}$ means that $\mathcal{A}$ is a subargument of $\mathcal{B}$ and $\mathcal{B}$ is a superargument of $\mathcal{A}$. Subarguments are arguments; therefore, every subargument belongs to $\mathbb{U}$, and they are a (distinguishable) part of the arguments they support. In this work, we will use the subargument concept to be able to eliminate some part of a given argument. The reason for this will be clear in the next section. We will refer to a *proper subargument* $\mathcal{A}_i$ of an argument $\mathcal{A}$ as $\mathcal{A}_i \sqsubset \mathcal{A}$, meaning that $\mathcal{A}_i \sqsubseteq \mathcal{A}$, but $\mathcal{A}_i \neq \mathcal{A}$. Finally, since the subargument relation is a partial order, it meets the properties of transitivity, antisymmetry, and reflexivity.

Definition 2 (Argument) *An* ***argument*** *is a set of interrelated pieces of knowledge supporting a claim from evidence and satisfying:* ***Self-Consistency:*** *$\mathcal{A}$ is self-consistent wrt.* **R** *iff there are no $\mathcal{A}_i \sqsubseteq \mathcal{A}$, $\mathcal{A}_j \sqsubseteq \mathcal{A}$ such that $\mathcal{A}_i \mathbf{R} \mathcal{A}_j$ nor $\mathcal{A}_j \mathbf{R} \mathcal{A}_i$.* ***Minimality:*** *$\mathcal{A}$ is minimal iff $\mathcal{A}$ supports α and there is no $\mathcal{A}_i \sqsubset \mathcal{A}$ such that $\mathcal{A}_i$ supports α.*

Definition 3 (Atomic Argument) *Let $\langle \mathbb{U}, \mathbb{A}, \mathbf{R}, \sqsubseteq \rangle$ be a DAF. An argument $\mathcal{A} \in \mathbb{U}$ is* ***atomic*** *iff there is no $\mathcal{B} \in \mathbb{U}$ such that $\mathcal{B} \sqsubset \mathcal{A}$.*

We will refer as *regular arguments* to those defined in the usual sense, that is, as a reason that supports a claim upon available evidence. We will consider *evidence* to be an active regular atomic argument, *i.e.*, a piece of evidence is an argument by itself. This association turns out to be quite natural, since pieces of evidence can be thought as indivisible and (self) conclusive. Note that an inactive regular atomic argument will not be considered as evidence. Atomic arguments will also allow us to identify the building blocks of an argument, *i.e.*, its minimal portions, as will be clear in Example 1.

Besides regular arguments, we will identify *potential arguments* as a supporting structure that is incomplete due to a lack of evidence (analogous to the concept introduced in [8]). Thus, although potential arguments have an associated claim, they cannot derive it by themselves. When evidence is not available, potential arguments will need claims from other arguments to be able to reach their own. Therefore, the set $\mathbb{U}$ can be seen as containing arguments that are not only active or inactive, but can be also regular or potential, and even atomic or not, thus dividing $\mathbb{U}$ in eight classes of argument. The need for three levels of classification will be clear in Section 3. For the definition of a potential argument the following functions are necessary:

Atomic: $\mu : \mathbb{U} \to \mathcal{P}(\mathbb{U})$ is such that $\mu(\mathcal{A}) = \{\mathcal{A}_i | \mathcal{A}_i$ is an atomic argument, $\mathcal{A}_i \sqsubset \mathcal{A}\}$

Completion: $\chi : \mathbb{U} \to \mathbb{U}$ where $\chi(\mathcal{A})$ is a regular argument $\mathcal{B}$ composed only by atomic arguments, such that $\mu(\mathcal{A}) \subset \mu(\mathcal{B})$, and $\mu(\mathcal{B}) \backslash \mu(\mathcal{A})$ is a non-empty set of regular atomic arguments.

There are multiple ways of completing any given argument, but the completion function χ gives the only one composed by the atomic arguments of $\mathcal{A}$ along with the necessary regular atomic arguments to support the claim.

Definition 4 (Potential Argument) *$\mathcal{A}$ is a* ***potential argument*** *for α iff $\mathcal{A}$ does not support α and the completion $\chi(\mathcal{A})$ is an argument for α (i.e., it satisfies minimality and self-consistency).*

In order for a potential argument to support its claim, other arguments should provide their own claims as a replacement for the missing evidence. Despite claims supported by an argument can be used when no evidence is available[1], they clearly differ in semantics: evidence is indisputable, whereas claims supported by an argument could be defeated. The notions of minimality and self-consistency indirectly apply to potential arguments, since their completion is a regular argument.

We will identify the set of regular arguments as **Reg** and the set of potential arguments as **Pot**. When convenient, these sets will be subscripted with $\mathbb{A}$ or $\mathbb{I}$, according to their nature of active or inactive. In what follows, we will use a graphical notation, intu-

[1] Note that a claim does not appear by itself, but it is always supported by an argument.

itive in nature, to depict regular and potential arguments: both will be represented as triangles, but potential arguments will contain a black "slot" in their base. The position of subarguments within an argument will suggest that upper potential subarguments need the claim of those placed at lower positions in order to reach a claim. This is shown in Figures 1 and 2, where subarguments at the base are required to reach a claim in order for the subargument at the top to reach its own.

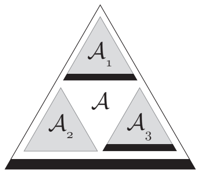
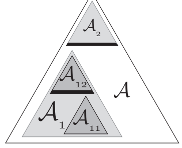


Figure 1. $\mathcal{A} \in$ **Pot**

For the following example only, we will assume a structure for arguments by using (propositional) logic programming. This will be useful to understand the graphical representation of arguments and subarguments and their condition of regular or potential.

Example 1

*Consider an argument $\mathcal{A} = \{c \leftarrow b, b \leftarrow a, a\}$, then we will say that $\mathcal{A}$ supports c, therefore $\mathcal{A} \in$ **Reg**. Moreover, the subargument $\mathcal{A}_1 = \{b \leftarrow a, a\}$ also belongs to **Reg** provided it supports b, but then the subargument $\mathcal{A}_2 = \{c \leftarrow b\}$ belongs to **Pot** given it cannot reach its claim c by itself. Furthermore, two subarguments $\mathcal{A}_{11}, \mathcal{A}_{12} \sqsubseteq \mathcal{A}_1$ are such that $\mathcal{A}_{11} = \{a\}$ and $\mathcal{A}_{12} = \{b \leftarrow a\}$, where the latter belongs to **Pot** and the former to **Reg**. This configuration is depicted in the figure. Notice that this graphical notation can replace the logical representation for arguments. From now on, we are going to rely on the graphical representation in order to make a complete abstraction over any underlying logic of arguments.*



The graphical representation allows us to recognize a particular configuration of subarguments: if a potential subargument is placed at the bottom of an argument triangle, then this argument is potential. For instance, in Figure 1 we have that argument $\mathcal{A}$ contains arguments $\mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_3$, where $\mathcal{A}_1$ is a potential argument that is fed by the claims of $\mathcal{A}_2$ and $\mathcal{A}_3$. Thus, since $\mathcal{A}_3$ is a potential subargument of $\mathcal{A}$ (placed at its base), then $\mathcal{A}$ is also potential. In opposition to this, in Figure 2 is shown a similar case, in which both arguments at the base of the triangle ($\mathcal{B}_2$ and $\mathcal{B}_3$) are not potential, and therefore the whole argument $\mathcal{B}$ is not potential.

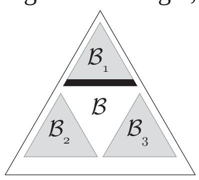
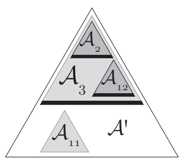


Figure 2. $\mathcal{B} \in$ **Reg**

Definition 5 (Equistructural Arguments) *Let μ be the Atomic function. Two arguments $\mathcal{A}_1$ and $\mathcal{A}_2$ are said to be* ***equistructural*** *iff $\mu(\mathcal{A}_1) = \mu(\mathcal{A}_2)$.*

Example 2

We can give an alternative representation $\mathcal{A}$' for argument $\mathcal{A}$ in Example 1. Since the sets of atomic arguments (i.e., $\mathcal{A}_{11}$, $\mathcal{A}_{12}$, $\mathcal{A}_2$) of $\mathcal{A}$ and $\mathcal{A}$' are the same, $\mathcal{A}$ and $\mathcal{A}$' are equistructural. Note that the order from bottom to top of the atomic arguments is the same in both cases, as would be expected.



Having more than one representation for the same argument could be a problem; in those situations, equistructurality would allow us to identify the class of equistructural arguments from which just one should be used.

As stated above, arguments can be either active or inactive. The interaction between arguments and subarguments regarding activeness will be made clear by the **activeness**

propagation principle, which defines the dynamics of the system. A group of arguments being active determines that an argument containing them (exclusively) is going to be active. Furthermore, a single argument becoming active makes all of its subarguments to become active. This also works the other way around: if a subargument $\mathcal{A}_i$ of an argument $\mathcal{A}$ is set inactive, then every superargument of $\mathcal{A}_i$ is set inactive, including $\mathcal{A}$. Moreover, the inactiveness of $\mathcal{A}_i$ means that at least one subargument of it is inactive. Activeness propagation is formally defined as follows:

(Activeness Propagation) $\mathcal{A} \in \mathbb{A}$ *iff* for every $\mathcal{A}_i \sqsubseteq \mathcal{A}$ we have that $\mathcal{A}_i \in \mathbb{A}$.

As a basis for our analysis we will use a DAF enriched with the notion of *dialectical constraint*. The resulting extended framework will be called a *dynamic argumentation theory*. The following definitions are an extended version of those in [9].

Definition 6 (Argumentation Line) *Let Φ be a DAF. An **argumentation line** λ in Φ is any finite sequence of arguments $[\mathcal{A}_1, \ldots, \mathcal{A}_n]$ such that $\mathcal{A}_i$ $\mathbf{R}$ $\mathcal{A}_{i-1}$, for $1 < i \leq n$. If $\mathcal{A}_1$ is the first element in λ, we will also say that λ is **rooted in** $\mathcal{A}_1$. The **upper segment** of λ wrt. $\mathcal{A}_i$, is defined as $\lambda^{\uparrow}(\mathcal{A}_{i>1}) = [\mathcal{A}_1, \ldots, \mathcal{A}_{i-1}]$, and $\lambda^{\uparrow}(\mathcal{A}_1)$ does not exist.*

Definition 7 (Set of Interference (Supporting) Arguments) *Let λ be an argumentation line, then the **set of interference** (resp., **supporting**)* arguments *λ^- (resp., λ^+) of λ is the set containing all the arguments placed on **even** (resp., **odd**) positions in λ.*

We will write $\mathfrak{Lines}_\Phi$ to denote the set of all possible argumentation lines regarding the arguments in Φ. These lines define a domain onto which different constraints can be defined. As such constraints are related to sequences which resemble an argumentation dialogue between two parties, we call them *dialectical constraints*. Formally:

Definition 8 (Dialectical Constraint) *Let Φ be a DAF. A **dialectical constraint** $\mathbf{C}$ in the context of Φ is any function $\mathbf{C} : \mathfrak{Lines}_\Phi \rightarrow \{True, False\}$. A given argument sequence $\lambda \in \mathfrak{Lines}_\Phi$ satisfies $\mathbf{C}$ in Φ when $\mathbf{C}(\lambda) = True$.*

Definition 9 (Dynamic Argumentation Theory) *A **dynamic argumentation theory** (DAT) T is a pair $(\Phi, \mathbf{DC})$, where Φ is a DAF satisfying activeness propagation, and $\mathbf{DC} = \{\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_k\}$ is a finite (possibly empty) set of dialectical constraints.*

Definition 10 (Acceptable Argumentation Line) *Given a DAT $T = (\Phi, \mathbf{DC})$, an argumentation line λ is **acceptable** wrt. T iff λ satisfies every $\mathbf{C}_i \in \mathbf{DC}$, and every $\mathcal{B} \in \lambda$ is active regular.*

In what follows, we will assume that the notion of acceptability imposed by dialectical constraints is such that if λ is acceptable wrt. a DAT $T = (\Phi, \mathbf{DC})$, then any subsequence of λ is also acceptable. We also assume a dialectical constraint that avoids the construction of circular argumentation lines, thus no line will contain two or more equistructural arguments.

Definition 11 (Bundle Set) *Given a DAT T, a set $S = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ of argumentation lines rooted in a given argument $\mathcal{A}$, denoted $S_{\mathcal{A}}$, is called a **bundle set** wrt. T iff there is no pair $\lambda_i, \lambda_j \in S_{\mathcal{A}}$ such that λ_j is a subsequence of λ_i.*

Definition 12 (Dialectical Tree) *Let T be a DAT, and let $\mathcal{A}$ be an argument in T and let $S_{\mathcal{A}} = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ be a bundle set. The* ***dialectical tree*** *rooted in $\mathcal{A}$ based on $S_{\mathcal{A}}$ (denoted $\mathcal{T}_{\mathcal{A}}$) is a tree-like structure defined as follows:*

1. *The root node of $\mathcal{T}_{\mathcal{A}}$ is $\mathcal{A}$.*
2. *Let $F = \{\mathsf{tail}(\lambda)$, for every $\lambda \in S_{\mathcal{A}}\}$, and $H = \{\mathsf{head}(\lambda)$, for every $\lambda \in F\}$.*[2]
   If $H = \emptyset$ then $\mathcal{T}_{\mathcal{A}}$ has no subtrees. Otherwise, if $H = \{\mathcal{B}_1, \ldots, \mathcal{B}_n\}$, then for every $\mathcal{B}_i \in H$, we define: $\text{getBundle}(\mathcal{B}_i) = \{\lambda \in F \mid \mathsf{head}(\lambda) = \mathcal{B}_i\}$
   We put $\mathcal{T}_{\mathcal{B}_i}$ as the immediate subtree or $\mathcal{A}$ based on $\text{getBundle}(\mathcal{B}_i)$.

We will denote $\mathfrak{Tree}_T$ to the family of all possible dialectical trees in the DAT T.

Acceptable dialectical trees are a subclass of dialectical trees that contain only acceptable argumentation lines. In the sequel, we will just write "dialectical trees" to refer to acceptable dialectical trees, unless stated otherwise. Acceptable dialectical trees allow to determine whether the root node of the tree is to be accepted (ultimately *undefeated*) or rejected (ultimately *defeated*) as a rationally justified belief. A *marking function* provides a definition of such acceptance criterion. Formally:

Definition 13 (Marking criterion) *Let T be a DAT. A* ***marking criterion*** *for T is a function* $\text{Mark} : \mathfrak{Tree}_T \to \{D, U\}$. *We will write* $\text{Mark}(\mathcal{T}_{\mathcal{A}}) = U$ *(resp.* $\text{Mark}(\mathcal{T}_{\mathcal{A}}) = D$*) to denote that the root node $\mathcal{A}$ of $\mathcal{T}_{\mathcal{A}}$ is marked as undefeated (resp. defeated).*

Definition 14 (Warrant) *Let T be a DAT and* Mark *a marking criterion for T. An active regular argument $\mathcal{A}$ is a* ***warranted argument*** *(or just* ***warrant****) wrt. a marking criterion* Mark *in T iff the dialectical tree $\mathcal{T}_{\mathcal{A}}$ is such that* $\text{Mark}(\mathcal{T}_{\mathcal{A}}) = U$.

Example 3

The digraph of arguments in Figure 3(a) describes a DAF, where the nodes are arguments and the arcs denote the attack relation with the arrowhead pointing to the argument under attack. The set $\mathbb{A}$ of active arguments is shown, along with the universal $\mathbb{U}$, and the set $\mathbb{I}$ of inactive arguments, which are illustrated as dashed triangles. Subarguments were drawn following the aforementioned convention. Note that the superargument of $\mathcal{A}$ is inactive because it has an inactive subargument.

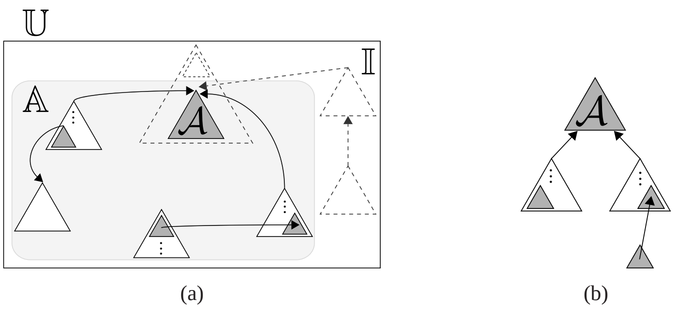


Figure 3. (a) Dynamic argumentation framework example (b) Tree spanning from $\mathcal{A}$

Figure 3(b) shows a dialectical tree spanning the graph from argument $\mathcal{A}$. Observe that despite an attack occurs between an inactive and an active argument, inactive ar-

[2]The functions $\mathsf{head}(\lambda)$ and $\mathsf{tail}(\lambda)$ have the usual meaning in list processing.

guments are not considered when analyzing the tree. The marking of this dialectical tree would allow us to determine if the root argument is warranted. Consider a marking function where each node of the tree is undefeated if either it is a leaf or all of its defeaters are defeated. With such a marking, the root node would be defeated. This status could be changed if we deactivate either the root's left defeater or both.

3. An Approach to an Argument Theory Change

We will briefly introduce some of the basic concepts of the belief revision theory [10]. Classic operations in the theory change, as those specified in the AGM model [11], are known as expansions, contractions, and revisions. An *expansion* adds a new belief to the epistemic state without guaranteeing its consistency after the operation. A *contraction* eliminates a belief from the epistemic state and some beliefs that make possible its deduction. The sentences to eliminate might represent the *minimal change* on the epistemic state. Finally, a *revision* inserts a sentence into the epistemic state, guaranteeing consistency if the input sentence was consistent. Hence, a revision adds a new belief possibly eliminating others to avoid inconsistencies. The latter change operation has been defined through the Levi Identity [6], which is a composition of sub-operations that ensures consistency by contracting the negation of the sentence at issue, and therefore by expanding it to the resulting knowledge base.

Regarding an argument change theory, a useful kind of revision would be to add an argument to a theory in such a way that this argument ends up being warranted. Therefore, in the rest of the article, we explore a contraction operator that will allow us to define a revision operator that follows the desired behavior. Firstly, we will introduce the basic theoretical elements required to modify a dialectical tree and turn the marking of its root argument to warranted. Then, we will define these three operations: (1) *Argument expansion*, which activates an argument; (2) *Non-warrant argument contraction*, which deactivates arguments in a particular tree looking to warrant its root; (3) *Warrant-prioritized argument revision* (WPA revision), defined from the previous operations.

3.1. Basic Theoretical Elements for Argument Theory Change

We need to characterize the kind of argumentation lines that actually affect the status of the root argument. We will call these lines *attacking lines*, and will be those over which the argument selection and then the argument incision are going to be applied. Although the main idea is to turn every attacking line into non-attacking, the correctness of the revision must not depend on whether it is possible to determine which lines are attacking; that is, if selections and incisions are applied to every line in the tree, the revision should remain correct. The condition of attacking for a given argumentation line is strictly dependant on the adopted marking function.

Definition 15 (Set of Attacking Lines) *Given a DAT T and the dialectical tree $\mathcal{T}_{\mathcal{A}}$ based on the bundle set $S_{\mathcal{A}}$, the* ***set of attacking lines*** $\mathrm{Att}_{\mathcal{A}}$ *over $\mathcal{A}$ is the minimal subset of $S_{\mathcal{A}}$ such that $S'_{\mathcal{A}} = S_{\mathcal{A}} \setminus \mathrm{Att}_{\mathcal{A}}$ is the bundle set for $\mathcal{A}$ in a hypothetical DAT T', where the dialectical tree based on $S'_{\mathcal{A}}$ warrants $\mathcal{A}$.*

Remark 1 (Notational Shortcuts) *In the rest of the article, we will work on a DAT* $T = (\Phi, \mathbf{DC})$, $\Phi = \langle \mathbb{U}, \mathbb{A}, \mathbf{R}, \sqsubseteq \rangle$ *with an active regular argument* $\mathcal{A}$ *as the root of a dialectical tree* $\mathcal{T}_{\mathcal{A}}$ *(from* $\mathfrak{Tree}_T$*) based on a bundle set* $S_{\mathcal{A}} \subseteq \mathfrak{Lines}_{\Phi}$*, where its (acceptable) argumentation lines* $\lambda_i \in S_{\mathcal{A}}$ *are to be associated to* $\mathcal{T}_{\mathcal{A}}$ *by the notation* $\lambda_i \in \mathcal{T}_{\mathcal{A}}$*. Arguments will be noted only as* $\mathcal{A}$*,* $\mathcal{B}$*, and* $\mathcal{C}$*, being* $\mathcal{A}$ *always the root of* $\mathcal{T}_{\mathcal{A}}$*, and* $\mathcal{B}$ *and* $\mathcal{C}$*, inner nodes in that tree, i.e.,* $\mathcal{A}, \mathcal{B}, \mathcal{C} \in \mathcal{T}_{\mathcal{A}}$*. Finally,* $\mathcal{B}_i^+$ *(resp.* $\mathcal{B}_i^-$*) will mean that* $\mathcal{B} \in \lambda_i^+$ *(resp.* $\mathcal{B} \in \lambda_i^-$*).*

Definition 16 (Argument Selection Function "γ") *An **argument selection function*** $\gamma : \mathfrak{Lines}_{\Phi} \rightarrow \mathbb{U}$ *is applied to every attacking line* $\lambda_i \in \mathrm{Att}_{\mathcal{A}}$ *in such a way that* $\gamma(\lambda_i) = \mathcal{B}_i^-$ *and* $\lambda_i^{\uparrow}(\mathcal{B}_i^-) \notin \mathrm{Att}_{\mathcal{A}}$*. In what follows, we will refer to the selected argument* $\mathcal{B}_i^-$ *just as* Ψ_i*.*

It is reasonable to require the argument selection function to return an interference argument, since these arguments are the ones that contradict the root. The condition of attacking of a given line should only depend on interference arguments –deactivating a supporting argument should not turn an attacking line into a non-attacking line. Furthermore, the deactivation of an interference argument from a non-attacking line should not turn this line into an attacking line.

Definition 17 (Argument Incision Function "σ") *A function* $\sigma : \mathbb{U} \rightarrow \mathcal{P}(\mathbb{U})$ *is an **argument incision function** iff* $\emptyset \subset \sigma(\Psi_i) \subseteq \mu(\Psi_i)$*.*

Incisions should be guided by an "intra-argument" criterion. For instance, they could be defined following some epistemic entrenchment method, namely, evidence might be considered more important than any other subargument. Therefore, it would be preserved from being cut off, unless it is not possible. The way arguments are incised should be defined by an internal selection function, which is out of the scope of this paper. Sometimes incisions will affect more arguments than the one being incised. In order to identify this situation, we introduce the notion of *collateral incision*; formally:

Definition 18 (Collateral Incision) *A **collateral incision** over* $\mathcal{B}_j$ *is defined as* $\sigma(\Psi_i) \cap \mu(\mathcal{B}_j) \neq \emptyset$*. If* $\sigma(\Psi_i) \cap \mu(\mathcal{C}_j) = \emptyset$ *for every* $\mathcal{C}_j \in \lambda_j^{\uparrow}(\mathcal{B}_j)$*, we will say that* $\sigma(\Psi_i)^{(\mathcal{B}_j)} = \sigma(\Psi_i) \cap \mu(\mathcal{B}_j)$ *is the **uppermost collateral incision**.*

Collateral incisions bring about some drawbacks: supporting arguments could be involuntarily deactivated, which might turn a non-attacking line into an attacking line. Besides, as said before, it is not reasonable to think that if the supporting argument belonged to an attacking line, its deactivation would change the line's status. Moreover, although a collaterally incised interference argument does not turn lines into attacking, it would also be an unnecessary incision. Therefore, it is desirable to select arguments in which any incision would never result in a collateral incision to other arguments. This is captured by the *cautiousness* property:

(Cautiousness) $\mu(\Psi) \cap \mu(\mathcal{B}) = \emptyset$, for any $\mathcal{B}$

Definition 19 (Cautious and Non-Cautious Selections) *A selection* Ψ *is identified as **cautious** iff it verifies **cautiousness**; otherwise, it is identified as **non-cautious**.*

Sometimes cautiousness may not be satisfied. In such a case, when a non-cautious selection is unavoidable, the incision in it should avoid any collateral incisions over any argument. However, this situation may not be always prevented and should be properly addressed. These difficulties are captured by the following principle:

(Preservation) *If* $\sigma(\Psi_i)^{(\mathcal{B}_j)} \neq \emptyset$ *then*
exists $\lambda_j^{\uparrow}(\mathcal{B}_j)$ *and* ($\Psi_j \in \lambda_j^{\uparrow}(\mathcal{B}_j)$ *iff* $\lambda_j^{\uparrow}(\mathcal{B}_j) \in \text{Att}_{\mathcal{A}}$),
for any $\mathcal{B}_j$

This principle is illustrated in Figure 4. When an incision $\sigma(\Psi_i)$ in the i^{th} dialectical line (the left branch in Figure 4) results in an uppermost collateral incision $\sigma(\Psi_i)^{(\mathcal{B}_j)}$ over argument $\mathcal{B}_j$ in the j^{th} dialectical line (right branch), it must be ensured that the selection Ψ_j in the j^{th} line is performed over the upper segment $\lambda_j^{\uparrow}(\mathcal{B}_j)$. This selection is only performed if $\lambda_j^{\uparrow}(\mathcal{B}_j)$ is an attacking line. Finally, note that if $\mathcal{B}_j$ is the root node, then there is no upper segment for it.

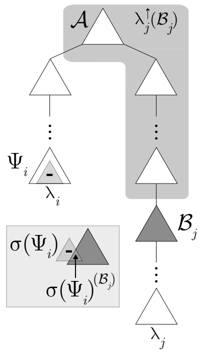


Figure 4. Preservation

In the case of the antecedent of the preservation principle being false (when there is no collateral incision over any argument $\mathcal{B}_j$ in any j^{th} line) the validity of the preservation principle is not threatened. This particular case may be referred as:

(Strict-Preservation) $\sigma(\Psi)^{(\mathcal{B})} = \emptyset$, for any $\mathcal{B}$

An incision satisfying strict-preservation ensures no argument is collaterally incised, although this principle cannot be always verified. The following two propositions address the relation between cautiousness and strict-preservation. Proposition 1.1 states that, when a selection is cautious, there is no overlapping with any argument; therefore, the incision over that selection verifies strict-preservation. However, a non-cautious selection could verify strict-preservation if, even though it overlaps with some argument, the incision over that selection is performed outside this overlapping. In this case, there is no collateral incision and strict-preservation holds (Proposition 1.2). Achieving strict-preservation regardless cautiousness may be also a desirable property.

Proposition 1 [3]
(1) If Ψ_i *is cautious, then* $\sigma(\Psi_i)$ *is strict-preserving.*
(2) If Ψ_i *is non-cautious and there exists* $\mathcal{C} \in \mu(\Psi_i)$ *such that* $\mathcal{C} \not\sqsubseteq \mathcal{B}_j$ *(for every* $\mathcal{B}_j$*), then there exists some* $\sigma(\Psi_i)$ *such that it is strict-preserving.*

Regarding collateral incisions, it is paramount to preserve the root argument $\mathcal{A}$ as active. In order to achieve this, no collateral incision should affect any subargument of $\mathcal{A}$; otherwise, it would be impossible to warrant it.

(Root-Preservation) $\sigma(\Psi)^{(\mathcal{A})} = \emptyset$

Root-preservation is a particular case of **strict-preservation**, where the argument $\mathcal{B}$ is the root argument $\mathcal{A}$. Since $\lambda_j^{\uparrow}(\mathcal{A})$ does not exist, the consequent of this instance

[3]Formal proofs were omitted due to space reasons.

of the preservation principle is always false, which means that the antecedent should be false in order for the principle to hold. This is so when root-preservation is satisfied. Therefore, collateral incisions over the root argument should always be avoided.

Proposition 2 *Regarding an argument incision function "σ", if preservation is satisfied, then root-preservation is also satisfied.*

Definition 20 (Warranting Incision Function) *An argument incision function "σ" verifying preservation is said to be a **warranting incision function**.*

In the following subsections, we will introduce the expansion and the non-warrant contraction operators in order to define the warrant-prioritized revision operator.

3.2. Argument Change Operators

The argument expansion can be defined in a simple manner by just adding an argument to the set of active arguments; formally:

Definition 21 (Argument Expansion) *An **argument expansion operator** "$+^{\triangle}$" over T by a regular argument $\mathcal{A} \in \mathbb{U}$, namely $T +^{\triangle} \mathcal{A}$, is defined as follows:*

$$T +^{\triangle} \mathcal{A} = (\langle \mathbb{U}, \mathbb{A} \cup \{\mathcal{A}\}, \mathbf{R}, \sqsubseteq \rangle, \mathbf{DC})$$

Note that whenever an argument $\mathcal{A}$ is activated, by activeness propagation every subargument in it is automatically activated. This is part of the dynamism of the theory. Moreover, the definition of the argument expansion has the inherent implications to expansions within any non-monotonic formalism: despite of the set $\mathbb{A}$ being increased, the amount of warranted consequences could be diminished.

An argument contraction operator could be defined analogously to this expansion by incising a given argument, thus deactivating it. Nonetheless, this contraction would not be very useful towards the definition of a warrant-prioritized revision operation. Next we will define a particular kind of contraction devoted to this purpose.

This contraction operator provides warrant for an argument $\mathcal{A} \in \mathbb{A}$ by turning every attacking line in $\mathcal{T}_{\mathcal{A}}$ to a non-attacking line through an argument incision function σ.

Definition 22 (Non-Warrant Argument Contraction) *A **non-warrant argument contraction operator** "$-^{\omega}$" of T by a regular argument $\mathcal{A} \in \mathbb{A}$, namely $T -^{\omega} \mathcal{A}$, is defined by means of a **warranting incision function** "σ" applied over selections $\Psi_i = \gamma(\lambda_i)$ for each $\lambda_i \in \mathrm{Att}_{\mathcal{A}}$ in $\mathcal{T}_{\mathcal{A}}$, as follows:*

$$T -^{\omega} \mathcal{A} = (\langle \mathbb{U}, \mathbb{A} \setminus \bigcup_i \sigma(\Psi_i), \mathbf{R}, \sqsubseteq \rangle, \mathbf{DC})$$

In contrast to the expansion, in the case of contractions the deactivation of atomic arguments by an incision involves the automatic deactivation of their superarguments.

A warrant-prioritized argument (WPA) revision operator should look for the expansion of an argument $\mathcal{A}$ revising the warrant condition of its claim. This means that after the argument expansion "$+^{\triangle}$" of $\mathcal{A}$, we should warrant its claim by effect of a non-warrant argument contraction "$-^{\omega}$". This operation is formally defined as follows:

Definition 23 (Warrant-Prioritized Argument Revision) *A **warrant-prioritized argument revision operator** of T by a regular argument $\mathcal{A} \in \mathbb{U}$, namely $T \times^{\omega} \mathcal{A}$, is defined by means of a **warranting incision function** "σ" applied over selections $\Psi_i = \gamma(\lambda_i)$ for each $\lambda_i \in \mathrm{Att}_{\mathcal{A}}$ in $\mathcal{T}_{\mathcal{A}}$, as follows:*

$$T \times^{\omega} \mathcal{A} = (\langle \mathbb{U}, (\mathbb{A} \cup \{\mathcal{A}\}) \setminus \bigcup_i \sigma(\Psi_i), \mathbf{R}, \sqsubseteq \rangle, \mathbf{DC})$$

Definition 23 can be rewritten in terms of an argument expansion and a non-warrant contraction as an analogy of the Reversed Levi Identity [10]:

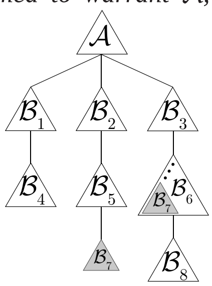
$$\textbf{(Argument Change Identity)} \quad T \times^{\omega} \mathcal{A} = (T +^{\triangle} \mathcal{A}) -^{\omega} \mathcal{A}$$

In order to warrant an argument $\mathcal{A}$ from a theory T, a non-warranting contraction is applied considering the tree $\mathcal{T}_{\mathcal{A}}$. If $\mathcal{A} \notin \mathbb{A}$, then $\mathcal{A}$ is impossible to warrant since $\mathcal{T}_{\mathcal{A}}$ does not exist. This means that, when revising T by $\mathcal{A}$, first we need to expand T by $\mathcal{A}$, thus assuring that $\mathcal{A}$ is active, and $\mathcal{T}_{\mathcal{A}}$ can be built. Therefore, the non-warranting contraction of $T +^{\triangle} \mathcal{A}$ by $\mathcal{A}$ can be performed, leading to the argument change identity.

Theorem 1 *Let T be a DAT, "$\times^{\omega}$", a WPA Revision Operator, and $\mathcal{A}$, an argument. If $T_R = T \times^{\omega} \mathcal{A}$ is the DAT revised by $\mathcal{A}$, then $\mathcal{A}$ is warranted from T_R.*
Proof sketch: Let $\mathcal{T}_{\mathcal{A}}$ be a tree from $(T +^{\triangle} \mathcal{A})$. If $\mathcal{A}$ is not warranted, then there is at least one attacking line (λ_i). Selections over attacking lines ($\gamma(\lambda_i)$) return an interference argument (Ψ_i) responsible for that line being attacking. Incisions over the selected arguments ($\sigma(\Psi_i)$) leave a subtree of $\mathcal{T}_{\mathcal{A}}$ containing non-attacking upper segments ($\lambda_i^{\uparrow}(\Psi_i)$) of the former attacking lines. If uppermost collateral incisions ($\sigma(\Psi_i)^{(\mathcal{B}_j)}$) occur and their upper segments ($\lambda_j^{\uparrow}(\mathcal{B}_j)$) turn out to be attacking lines, they will be considered in concordance with the preservation principle. Finally, the tree resulting from the selection and incision process contains no attacking lines, and therefore $\mathcal{A}$ is warranted.

Example 4

Let us consider a non-warrant argument contraction performed to warrant $\mathcal{A}$, whose tree is depicted in the figure. In this example we will select the lowest possible argument satisfying cautiousness within each attacking line in the tree. This criterion attempts to preserve the tree structure. Besides, we will use the marking function from Example 3, and assume that attacking lines are those ending with an interference argument. Regarding the line $[\mathcal{A}, \mathcal{B}_1, \mathcal{B}_4]$, no incision has to be performed, since it ends with a supporting argument. Line $[\mathcal{A}, \mathcal{B}_2, \mathcal{B}_5, \mathcal{B}_7]$ is attacking and $\mathcal{B}_2$ should be selected, since selecting $\mathcal{B}_7$ would violate cautiousness. Finally, in the line $[\mathcal{A}, \mathcal{B}_3, \mathcal{B}_6, \mathcal{B}_8]$, $\mathcal{B}_8$ is selected and incised. The resulting argumentation lines after the contraction are $[\mathcal{A}, \mathcal{B}_1, \mathcal{B}_4]$ and $[\mathcal{A}, \mathcal{B}_3, \mathcal{B}_6]$, and $\mathcal{A}$ ends up warranted.



In the hypothetical case of a selection choosing the lowest argument regardless cautiousness, the selection in line $[\mathcal{A}, \mathcal{B}_2, \mathcal{B}_5, \mathcal{B}_7]$ would be $\mathcal{B}_7$. Note that its incision would inevitably affect $\mathcal{B}_6$, which ends up deactivated by a collateral incision. The upper segment of $\mathcal{B}_6$ in line $[\mathcal{A}, \mathcal{B}_3, \mathcal{B}_6, \mathcal{B}_8]$ is $[\mathcal{A}, \mathcal{B}_3]$, which is an attacking line, and thereafter argument $\mathcal{B}_3$ is selected and later on incised, because of preservation. The resulting lines after this contraction are $[\mathcal{A}, \mathcal{B}_1, \mathcal{B}_4]$ and $[\mathcal{A}, \mathcal{B}_2, \mathcal{B}_5]$ and $\mathcal{A}$ is thus warranted.

The latter case shows why the property of preservation is needed: otherwise argument $\mathcal{B}_8$ could be eligible to be incised, thus leaving the upper segment of $\mathcal{B}_6$ (i.e., $[\mathcal{A}, \mathcal{B}_3]$) as an attacking line for a defeated root argument.

4. Conclusions & Future Work

Throughout this paper, an abstract argumentation framework was proposed to be capable of dealing with knowledge dynamics. We also gave structure to arguments through the subargument relation without losing the property of being abstract. Along with this structure we defined an incomplete form of argument that can be put together with evidence in order to form an argument in the usual sense. To characterize the dynamics of the theory, we have shown how to adapt elements from the classic theory change to fit into the description of the proposed framework. The methods here introduced would be useful for an argumentation-based agent that is immersed in a changing environment.

Further analysis of the argument change operators was left as future work, including the specification of other versions of them and the definition of a set of basic postulates. Future work also includes the definition of change operators that works over the set of attack relations among arguments. This would add greater flexibility to the approach here presented, allowing for the representation of a dynamic preference criterion among arguments. Preferences could change either towards a goal or in response to a change in the "rules of the game". A similar idea could be applied to the set of dialectical constraints. Finally, the complex composition of arguments from their subarguments, along with their multiple representations, requires further study in order to define new properties for this theory such as theory equivalence and minimal theories.

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