Model-Based Contractions for Description Logics

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Abstract

Model-Based Contractions are a formalism –based on model-theoretic semantics– which characterizes an operation that modifies a knowledge base to avoid the satisfiability of a given expression. In the context of ontology revision, model-based contractions are a functional component that yields an ontology ready to evolve consistently. In this work we formalize the theory for contractions providing a model, variations, and their axiomatic characterization. Afterwards an algorithm towards its realization is proposed. Such algorithm has no further impact in computability, since it works on top of the satisfiability checking of the incoming information.

Introduction

We propose an ontology change operator which models the dynamics of the knowledge represented by ontologies. Such operator allow the addition of axioms (terminological descriptions) and assertions to the respective ontology in a consistent manner. In this sense, we follow consistency by assuming the change operator to be applied to ontologies for which consistency is a critical matter due to either, the domain they model, or the systems referring to them. Because of its highly reusable distributed nature, this kind of ontologies should pass only through consistent intermediate states of an evolutional process.

For an ontology change operation, it is important to follow the minimal change principle. This is related to the avoidance of instance data loss, *i.e.*, assertions, whenever it is possible. When some change is stated, as a result of that, some axioms may end up unsatisfiable, turning the ontology to incoherency. Here another important issue is unveiled regarding how the old and the new information is considered.

For the sake of ontology evolution, the new information is some new appreciation of the world produced by a change in the shared domain, or in its conceptualization. This means that the new knowledge should be prioritized over the older. Therefore, when some axiom experiences satisfiability loss it should be considered outdated wrt. the current state of the shared domain. Hence, its condition is analyzed in order to automatically restore its satisfiability, unless the ontology engineer interprets it needs to be treated separately.

Therefore, realization of the ontology change operation will rely on two sub-operations, the first one, namely *model-based contractions*, will modify the ontology accordingly to the incoming information such that it could be consistently and coherently incorporated later, whereas the latter sub-operation[1] will restore the satisfiability of the outdated axioms in order to coherently reincorporate them along with the new information in a consistent manner.

In this work, we formally define the model-based contraction operator, providing its axiomatic characterization, and an algorithm for its realization. Consequently, as description logic reasoners usually deal with huge ontologies, it is of utmost importance to provide an algorithm capable of reusing previous computations. This means that such algorithm should work on top of the satisfiability checking of the incoming information.

Description Logics Brief Overview

The following constitutes a very brief overview of the description logics (DLs) used in this paper, for more detailed information refer to (Baader *et al.* 2003). An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a nonempty domain $\Delta^{\mathcal{I}}$, and an interpretation function $\cdot^{\mathcal{I}}$ that maps every concept to a subset of $\Delta^{\mathcal{I}}$, every role to a subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, and every individual to an element of $\Delta^{\mathcal{I}}$.

The basic description language $\mathcal{AL}$ is formed by concept descriptions according to the syntax $C, D \longrightarrow A|\bot|\top|\neg A|C \sqcap D|\forall R.C|\exists R.\top$ where A is an atomic concept, R is an atomic role; and the interpretation function $\cdot^{\mathcal{I}}$ is extended to the universal concept as $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$; the bottom concept as $\bot^{\mathcal{I}} = \emptyset$; the atomic negation as $(\neg A)^{\mathcal{I}} = \Delta^{\mathcal{I}} \backslash A^{\mathcal{I}}$; the intersection as $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$; the universal quantification as $(\forall R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} | \forall b.(a,b) \in R^{\mathcal{I}} \to b \in C^{\mathcal{I}}\}$; and the limited existential quantification as $(\exists R.\top)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} | \exists b.(a,b) \in R^{\mathcal{I}}\}$.

More expressive languages are possible by adding different constructors to $\mathcal{AL}$ like union of concepts (identified as $\mathcal{U}$), interpreted as $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$; full existential quantification ($\mathcal{E}$), interpreted as $(\exists R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} | \exists b.(a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}$; full negation or complement ($\mathcal{C}$), interpreted as $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \backslash C^{\mathcal{I}}$; and more. Extend-

[1]The formalizations for the second sub-operation and the ontology change operation are out of the scope of this work.

ing $\mathcal{AL}$ by any of the above yields a particular language respectively named by a string of the form $\mathcal{AL}[\mathcal{U}][\mathcal{E}][\mathcal{C}]$. DLs considered in this work follow such kind of specifications.

A knowledge base (KB) is a pair $\Sigma = \langle \mathcal{T}_\Sigma, \mathcal{A}_\Sigma \rangle$, where $\mathcal{T}_\Sigma$ represents the TBox, containing the terminologies (or axioms) of the application domain, and $\mathcal{A}_\Sigma$, the ABox, which contains assertions about named individuals in terms of these terminologies. Regarding the TBox $\mathcal{T}_\Sigma$, axioms are sketched as $C \sqsubseteq D$ and $C \equiv D$, therefore, an interpretation $\mathcal{I}$ satisfies them whenever $C^\mathcal{I} \subseteq D^\mathcal{I}$ and $C^\mathcal{I} = D^\mathcal{I}$ respectively. An interpretation $\mathcal{I}$ is a model for the TBox $\mathcal{T}_\Sigma$ if $\mathcal{I}$ satisfies all the axioms in $\mathcal{T}_\Sigma$. Thus, the TBox $\mathcal{T}_\Sigma$ is said to be satisfiable if it admits a model. Besides, in the ABox $\mathcal{A}_\Sigma$, $\mathcal{I}$ satisfies $C(a)$ if $a \in C^\mathcal{I}$, and $R(a,b)$ if $(a,b) \in R^\mathcal{I}$. An interpretation $\mathcal{I}$ is said to be a model of the ABox $\mathcal{A}_\Sigma$ if every assertion of $\mathcal{A}_\Sigma$ is satisfied by $\mathcal{I}$. Hence, the ABox $\mathcal{A}_\Sigma$ is said to be satisfiable if it admits a model. Finally, regarding the entire KB, an interpretation $\mathcal{I}$ is said to be a model of Σ if every statement in Σ is satisfied by $\mathcal{I}$, and Σ is said to be satisfiable if it admits a model.

In the rest of this article, we write $\mathcal{L}$ to identify some specific $\mathcal{AL}[\mathcal{U}][\mathcal{E}][\mathcal{C}]$ DL. When necessary, we will make difference in a logic $\mathcal{L}$ between the representation for axioms, writing $\mathcal{L}_\mathcal{T}$, and the $\mathcal{L}$ logic for assertions, noted as $\mathcal{L}_\mathcal{A}$. A KB contains implicit knowledge that is made explicit through inferences. The notion of semantic entailment is given by $\Sigma \models \alpha$, meaning that every model of the KB $\Sigma \subseteq \mathcal{L}_\mathcal{T} \times \mathcal{L}_\mathcal{A}$ is also a model of the sentence $\alpha \in \mathcal{L}$.

(Semantic Entailment) $\Sigma \models \alpha$ *iff* $\mathcal{M}(\Sigma) \subseteq \mathcal{M}(\{\alpha\})$

The semantics of an ABox in DLs (as every ontological language), are characterized by the open world assumption (OWA), this means that absence of information in an ABox means nothing but lack of knowledge, in contrast to instances in databases where absence of information is interpreted as negative information.

Remark 1 (Restriction Towards a Practical Approach) *For some description languages (like $\mathcal{ALC}$), every satisfiable KB is known to have infinitely many models most of which are infinite. In order to provide a practical approach, we will restrict this work to finite sets of finite models. Besides, unique name assumption (UNA) is also assumed in order to assure that each individual (in the world) will map to a unique individual name. Finally, we will assume that the representation of every KB taken into consideration is made with an acyclic TBox.*

Given the assumptions above, a query α to the KB Σ, noted as $\Sigma \overset{?}{\models} \alpha$, is solved by checking if every element $\mathcal{I} \in \mathcal{M}(\Sigma)$ (*i.e.*, every model of Σ), is also a model of α. If this is true, the query is said to be satisfied, namely $\Sigma \models \alpha$, and α inferred, being `YES` the answer. To the contrary, if $\Sigma \models \neg\alpha$, the query is not satisfied, being `NO` its answer. Finally, if none of the previous are verified, *i.e.*, $\Sigma \not\models \alpha$ and $\Sigma \not\models \neg\alpha$, then the query is answered as `UNKNOWN`.

Intuitions for Ontology Change Operations

Given a DL $\mathcal{L}$, a consistent KB $\Sigma \subseteq \mathcal{L}_\mathcal{T} \times \mathcal{L}_\mathcal{A}$ and a satisfiable sentence $\varphi \in \mathcal{L}$, we want Σ to evolve towards a new KB $\Sigma^R \subseteq \mathcal{L}_\mathcal{T} \times \mathcal{L}_\mathcal{A}$ such that $\Sigma^R \models \varphi$, or equivalently, by reduction to unsatisfiability, $\Sigma^R \cup \{\neg\varphi\}$ has no models. An intuitive solution is to avoid the inference of $\neg\varphi$ such that the further inclusion of φ would end up in a consistent KB. Therefore, this change operation would be composed by two main sub-operations, passing from satisfaction of $\neg\varphi$, to uncertainty, and afterwards to satisfaction of φ. From a model-theoretic viewpoint (see Fig. 1), this is analogous to pass from a state in which every model satisfies $\neg\varphi$, through an intermediate state in which some models does not satisfy $\neg\varphi$, to a final state in which no model satisfies $\neg\varphi$, exactly as reduction to satisfiability requires.

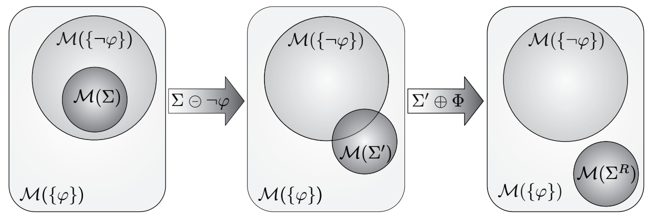

Figure 1: The operation $\mathfrak{C}^\Sigma(\varphi)$ and its two sub-operations.

As stated in the introductory section, we want to generate consistent outcomes from each sub-operation, preserving coherency and consistency of the final evolved KB. In this sense, as an effect of the first sub-operation, if an axiom is predicted to become unsatisfiable –considered along with the incoming sentence φ– it would be eliminated by the first sub-operation. Afterwards, the second sub-operation would repair such axioms and incorporate them along with φ to the resultant KB.

Satisfiability restoration of a given axiom may be achieved by considering its maximal consistent fragment. For this matter, we can take advantage of the advances achieved in the last years in the area of ontology debugging (Schlobach & Cornet 2003; Kalyanpur *et al.* 2006).

After the first sub-operation is applied, a second sub-operation consistently adds a set $\Phi = \{\varphi\} \cup \mathcal{T}_\mathcal{Z}$, where φ stands for the original expression, and $\mathcal{T}_\mathcal{Z}$, for the set of repaired axioms eliminated from Σ. Finally, the *ontology change operation* would be: $\mathfrak{C}^\Sigma(\varphi) = (\Sigma \ominus \neg\varphi) \oplus \Phi$, where "$\ominus$" refers to the first sub-operation, namely *model-based contraction*. The second sub-operation "$\oplus$" is out of the scope of this paper, and is part of our future work to formalize the specification of the *ontology change operator* $\mathfrak{C}$.

The following example shows, in an intuitive manner, how the ontology change operation would operate.

Example 1 *Let $\Delta^\mathcal{I} = \{a\}$ be a domain for the KB $\Sigma = \langle\{C \sqsubseteq D_1,\ D_3 \sqsubseteq C \sqcap \neg D_2\},\ \{C(a), D_1(a),\ \neg D_2(a), D_3(a)\}\rangle$. We want to consistently integrate the axiom $C \sqsubseteq D_1 \sqcap D_2$ to the KB. Since $C \sqsubseteq D_1$ is part of the TBox, the sentence φ to incorporate will be just $C \sqsubseteq D_2$. Thus, by effect of an ontology change operation $\mathfrak{C}^\Sigma(C \sqsubseteq D_2)$, we want to achieve a KB Σ^R such that $\Sigma^R \models \varphi$.*

By reduction to unsatisfiability, $\Sigma^R \cup \{(C \sqcap \neg D_2)(x)\}$ for a free variable x, should have no models. Consequently, *we will first generate a contracted KB $\Sigma' = \Sigma \ominus (C \sqcap \neg D_2)(x)$ such that $\Sigma' \models \neg\varphi$ is not verified. Two minimal proofs*[2] *arise for $\neg\varphi$: $\{C(a), \neg D_2(a)\}$ and $\{D_3 \sqsubseteq C \sqcap \neg D_2, D_3(a)\}$. Suppose now $\neg D_2(a)$ is retired from the former set, and the axiom $D_3 \sqsubseteq C \sqcap \neg D_2$ from the latter – given that it becomes unsatisfiable if considered along with $C \sqsubseteq D_2$. Hence, after the contraction "$\ominus$", the resultant KB would be $\Sigma' = \langle\{C \sqsubseteq D_1\}, \{D_1(a), C(a), D_3(a)\}\rangle$.*

From the second stage of the ontology change operation "$\mathfrak{C}$", the sentence $D_3 \sqsubseteq C \sqcap \neg D_2$ needs to be repaired. A possible solution may be to change it to $D_3 \sqsubseteq C$.[3] *Now, by effect of the final sub-operation "$\oplus$", we can consistently add the sentence φ along with the repaired axiom. Hence, the final KB would be $\Sigma^R = \langle\{C \sqsubseteq D_1 \sqcap D_2, D_3 \sqsubseteq C\}, \{D_1(a), C(a), D_3(a)\}\rangle$.*

Model-Based Contractions

Belief bases are sets of formulae not closed under logical consequence, containing implicit beliefs that are made explicit through inferences. Conforming the AGM model (Alchourrón, Gärdenfors, & Makinson 1985) of theory change, Kernel Contractions (Hansson 1994) are a construction for contracting belief bases based on the following intuition: in order to avoid the KB to infer a given sentence α, at least one element of each α-kernel (minimal α proof) is removed. Afterwards, no proof for α will appear in the resultant KB.

Applying directly kernel contractions to ontology languages –considering *open-world semantics*– important concerns appear. For instance, kernel contractions are not designed to reason following case analysis as done by semantic entailment. This problem is made clear below.

Example 2 *Let $\Sigma = \langle\emptyset, \{SU(a,b), SU(b,c), SU(c,d), Re(a), \neg Re(c), \neg Re(d)\}\rangle$ be a KB where SU and Re stand for "supervised-by" and "researcher", respectively. Now suppose a new regulation poses that no academic researcher might be supervised by a non-researcher. Therefore, we would need to provoke the ontology to evolve by an operation $\mathfrak{C}^{\Sigma}(\varphi)$, where $\varphi = Re \sqsubseteq \forall SU.Re$. In consequence, we should first apply a model-based contraction $\Sigma \ominus \alpha$, where $\alpha = (Re \sqcap \exists SU.\neg Re)(x)$, avoiding any certainty about the existence of some researcher who is supervised by a non researcher.*

Assuming a domain $\Delta^{\mathcal{I}} = \{a, b, c, d\}$, the only possibility to answer α is by case analysis on the interpretation $\mathcal{I}$ of the individual name b regarding the concept Re. That is, in case $b \in Re^{\mathcal{I}}$, the related minimal proof would be $\{Re(b), SU(b,c), \neg Re(c)\}$, whereas if $b \in \neg Re^{\mathcal{I}}$, the related minimal proof would be $\{Re(a), SU(a,b), \neg Re(b)\}$. Finally, the query $\Sigma \models \alpha$ is satisfied.

Queries in such situations cannot be correctly answered if case analysis is not performed. In such a case, kernel contractions would not recognize any α-kernel and therefore, the inference of α could not be avoided. An alternative for it could be its redefinition by use of the semantic entailment "$\models$", but again some new problems would appear since it would no longer find minimal proofs but minimal sets. Each of those sets will contain an incomplete proof considering an individual $a \in C^{\mathcal{I}}$ and the opposite proof for the case in which $a \in \neg C^{\mathcal{I}}$.

Example 3 (Ex. 2 cont.) *A set $\mathcal{K} = \{SU(a,b), SU(b,c), Re(a), \neg Re(c)\}$ is a minimal subset $\mathcal{K} \subseteq \Sigma$ entailing α, i.e., $\Sigma \models \alpha$.*

Example 4 *Given $\Sigma = \langle\emptyset, \mathcal{A}_\Sigma\rangle$, where $\mathcal{A}_\Sigma = \{SU(a,b), SU(b,c), SU(c,d), SU(e,b), SU(b,f), Re(a), Re(e), \neg Re(c), \neg Re(d), \neg Re(f)\}$, and the sentence $\alpha = (Re \sqcap \exists SU.\neg Re)(x)$. Considering a domain $\Delta^{\mathcal{I}} = \{a, b, c, d, e, f\}$, four different minimal sets for α appear:*

$\mathcal{K}_1 = \{SU(a,b), SU(b,c), Re(a), \neg Re(c)\}$
$\mathcal{K}_2 = \{SU(a,b), SU(b,f), Re(a), \neg Re(f)\}$
$\mathcal{K}_3 = \{SU(e,b), SU(b,c), Re(e), \neg Re(c)\}$
$\mathcal{K}_4 = \{SU(e,b), SU(b,f), Re(e), \neg Re(f)\}$

From the previous examples, in order to avoid $\Sigma \models \alpha$ we may eliminate at least one belief from each set $\mathcal{K}$ such that no minimal set for α would exist. Although this method "seems to successfully[4]" achieve a contraction operation, it is not so reasonable since this would require exponential space to compute. That is, $O(m^n)$, where m is an average of minimal α-proofs valid in each model $\mathcal{I} \in \mathcal{M}(\Sigma)$, and n is the cardinality of $\mathcal{M}(\Sigma)$. Hence, adapting the classical theory of kernel contractions to ontology languages, by simply changing its classical consequence operator "$\vdash$" to semantic entailment, seems to be quite unnatural and very inefficient.

Contractions Machinery

Our interest relies on a construction requiring polynomial space to compute. That is, a different construction is needed over which some new methodology is to be applied. This construction should rely on the KB and some sole model. In consequence, the methodology would run in polynomial space wrt. to the number of minimal α-proofs appearing in the construction determined by an appropriate model. Such an approach would allow to operate the change in a more efficient, intuitive, and natural manner; irrespective of considering a possibly infinite set $\mathcal{M}(\Sigma)$.

In general, our proposal is based on a similar intuition to that of Kernels, but since semantic entailment considers an inference by verifying its satisfiability in every model, minimal proofs for α should be found in every "extension" of the KB Σ. Such KB extension, namely $\Sigma_{\mathcal{I}}$, will contain the KB Σ extended by the assumptions made in the related model $\mathcal{I} \in \mathcal{M}(\Sigma)$.

Definition 1 (KB Extension "$\Sigma_{\mathcal{I}}$") *Let $\mathcal{L}$ be a DL, $\Sigma \subseteq \mathcal{L}_{\mathcal{T}} \times \mathcal{L}_{\mathcal{A}}$, a knowledge base such that $\Sigma = \langle\mathcal{T}_\Sigma, \mathcal{A}_\Sigma\rangle$, and $\mathcal{M}(\Sigma)$, its set of models. The finite Σ extension by a finite model $\mathcal{I} \in \mathcal{M}(\Sigma)$ is a KB $\Sigma_{\mathcal{I}} = \langle\mathcal{T}_\Sigma, \mathcal{A}_\Sigma \cup \mathrm{S}_{\Sigma_{\mathcal{I}}}\rangle$ where:*

$$\mathrm{S}_{\Sigma_{\mathcal{I}}} = \{C(a) | \forall a \in \Delta^{\mathcal{I}}, \textit{ where } a \in C^{\mathcal{I}} \textit{ and } C(a) \notin \mathcal{A}_\Sigma\} \cup \{R(a,b) | \forall a, b \in \Delta^{\mathcal{I}}, \textit{ where } (a,b) \in R^{\mathcal{I}} \textit{ and } R(a,b) \notin \mathcal{A}_\Sigma\}.$$

[2] Minimal proofs will be formally defined later.

[3] A brief discussion on this matter is given by the end of this article.

[4] It will be clear later that such methodology does not guarantee success.

The set $\mathrm{S}_{\Sigma_\mathcal{I}}$ is referred as "set of assumptions" determined by the finite model $\mathcal{I}$.

From the definition above, the extended ABox $\mathcal{A}_{\Sigma_\mathcal{I}}$ of $\Sigma_\mathcal{I}$ is composed by two different kinds of assertions: factual assertions, *i.e.*, those contained in $\mathcal{A}_\Sigma$; and non-factual assertions, or so called set of assumptions $\mathrm{S}_{\Sigma_\mathcal{I}}$.

Whenever $\Sigma \models \alpha$, for a KB Σ and a sentence α, we may identify different explanations for α conformed as minimal sets, inside some KB extension from a model $\mathcal{I}$.

Definition 2 (Extended α-Kernel "EαK") *Given a DL $\mathcal{L}$, a KB $\Sigma \subseteq \mathcal{L}_\mathcal{T} \times \mathcal{L}_\mathcal{A}$, its extension $\Sigma_\mathcal{I} = \langle \mathcal{T}_\Sigma, \mathcal{A}_\Sigma \cup \mathrm{S}_{\Sigma_\mathcal{I}} \rangle$ and a sentence $\alpha \in \mathcal{L}$. An extended α-Kernel (for short EαK), is a KB $\mathcal{K} = \langle \mathcal{T}_\mathcal{K}, \mathrm{F}_\mathcal{K} \cup \mathrm{S}_\mathcal{K} \rangle$ verifying the following conditions:*

(1) $\mathcal{K} \subseteq \Sigma_\mathcal{I}$, *i.e.*, $\mathcal{T}_\mathcal{K} \subseteq \mathcal{T}_\Sigma$, $\mathrm{F}_\mathcal{K} \subseteq \mathcal{A}_\Sigma$, *and* $\mathrm{S}_\mathcal{K} \subseteq \mathrm{S}_{\Sigma_\mathcal{I}}$.

(2) $\mathcal{K} \models \alpha$.

(3) *There is no $\mathcal{K}' \subset \mathcal{K}$ such that $\mathcal{K}' \models \alpha$.*

From Def. 2, given an EαK $\mathcal{K} = \langle \mathcal{T}_\mathcal{K}, \mathcal{A}_\mathcal{K} \rangle$, the factual assertions $\mathrm{F}_\mathcal{K} \subseteq \mathcal{A}_\mathcal{K}$ may be identified as $\mathcal{A}_\Sigma \cap \mathcal{A}_\mathcal{K}$. The set containing every EαK in a KB extension is defined as follows.

Definition 3 (Set of EαKs "$\Sigma_\mathcal{I} {\perp\!\!\!\perp} \alpha$") *Given a DL $\mathcal{L}$, a KB $\Sigma \subseteq \mathcal{L}_\mathcal{T} \times \mathcal{L}_\mathcal{A}$, and a sentence $\alpha \in \mathcal{L}$. The set $\Sigma_\mathcal{I} {\perp\!\!\!\perp} \alpha \subseteq 2^{\mathcal{L}_\mathcal{T} \times \mathcal{L}_\mathcal{A}}$ is the set of every EαK in the KB extension $\Sigma_\mathcal{I}$.*

Different models may determine different sets of EαKs for each associated KB extension, as is shown below.

Example 5 (Ex. 4 cont.) *Consider $\mathcal{I} \in \mathcal{M}(\Sigma)$ the model generating the KB extension $\Sigma_\mathcal{I} = \langle \mathcal{T}_\Sigma, \mathcal{A}_\Sigma \cup \mathrm{S}_{\Sigma_\mathcal{I}} \rangle$, where $\mathrm{S}_{\Sigma_\mathcal{I}} = \{\}$. Therefore, the related EαKs are the four $\mathcal{K}_i \subseteq \Sigma_\mathcal{I}$ detailed in Ex. 4. Later on, $\Sigma_\mathcal{I} {\perp\!\!\!\perp} \alpha = \{\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, \mathcal{K}_4\}$.*

Consider now, a different model $\mathcal{I}' = (\Delta^{\mathcal{I}'}, \cdot^{\mathcal{I}'})$, where $\Delta^{\mathcal{I}'} = \Delta^\mathcal{I}$, and $b \in \neg Re^\mathcal{I}$. Then, for the related KB extension $\Sigma_{\mathcal{I}'} = \langle \mathcal{T}_\Sigma, \mathcal{A}_\Sigma \cup \mathrm{S}_{\Sigma_{\mathcal{I}'}} \rangle$, it follows $\mathrm{S}_{\Sigma_{\mathcal{I}'}} = \{\neg Re(b)\}$. Finally, $\Sigma_{\mathcal{I}'} {\perp\!\!\!\perp} \alpha = \{\{Re(a), SU(a,b), \neg Re(b)\}, \{Re(e), SU(e,b), \neg Re(b)\}\}$.

We identify as *complete EαK* to an EαK containing all the necessary knowledge to infer α with no need to make case analysis, *i.e.*, no extra assumptions are required to be done. From the example above, each EαKin $\Sigma_{\mathcal{I}'} {\perp\!\!\!\perp} \alpha$ is complete.

We look for a KB extension in which every complete EαK is contained. In this sense, we can apply a methodology running in polynomial space wrt. to the number of EαKs in the KB extension. It is clear that not every model allows to determine such a KB extension. Thus, it is needed to extend the KB regarding some appropriate model $\mathcal{I}$.

In this sense, the appropriate KB extension may be determined by upper and lower bounds. That is, the KB extension should contain only the necessary non-factual assertions to (completely) explain α (*i.e.*, the upper bound); but in the other hand, every EαK should be completed using only non-factual assertions from the KB extension (*i.e.*, the lower bound). In the same manner, the model $\mathcal{I}$ determining such a KB extension should also consider an appropriate minimal domain. That is, the smallest domain which suffices to identify every complete EαK.

Definition 4 (α-Minimal Extension & α-Minimal Model) *Given a sentence α, a KB Σ, and a model $\mathcal{I} \in \mathcal{M}(\Sigma)$. The KB extension $\Sigma_\mathcal{I} = \langle \mathcal{T}_\Sigma, \mathcal{A}_\Sigma \cup \mathrm{S}_{\Sigma_\mathcal{I}} \rangle$ is α-minimal iff it follows:*

(1) $\mathrm{S}_{\Sigma_\mathcal{I}} = \bigcup_{\mathcal{K} \in (\Sigma_\mathcal{I} {\perp\!\!\!\perp} \alpha)} \mathrm{S}_\mathcal{K}$, *and*

(2) *there is no $\mathcal{I}' \in \mathcal{M}(\Sigma)$, where $\Delta^\mathcal{I} \subseteq \Delta^{\mathcal{I}'}$, such that for the related KB extension $\Sigma_{\mathcal{I}'}$ it holds (1) and $\Sigma_\mathcal{I} \subset \Sigma_{\mathcal{I}'}$.*

Consequently, the α-minimal model $\mathcal{I}$ is noted as $\mathcal{I}^\alpha$, and the associated α-minimal extension $\Sigma_\mathcal{I}$ is referred as $\Sigma_{\mathcal{I}^\alpha}$.

Proposition 1 *If $(\Sigma_{\mathcal{I}^\alpha} \subseteq \Sigma_\mathcal{I})$ then $(\Sigma_{\mathcal{I}^\alpha} {\perp\!\!\!\perp} \alpha = \Sigma_\mathcal{I} {\perp\!\!\!\perp} \alpha)$.*

In particular, we are interested in the set of EαKs "$\Sigma_{\mathcal{I}^\alpha} {\perp\!\!\!\perp} \alpha$" determined by an α-minimal model $\mathcal{I}^\alpha$. In the following example it is shown how the structure proposed so far is built. Note that, although more than one model is considered in the examples (and thus, more than one KB extension), the theory requires just one single model.

Example 6 *Consider $\Sigma = \langle \{C \sqsubseteq D\}, \{C(a), D(b)\} \rangle$, $\alpha = D(a)$, and models of domain $\{a, b\}$, determining: $\mathrm{S}_{\mathcal{I}_1} = \{C(b), D(a)\}$, and $\mathrm{S}_{\mathcal{I}_2} = \{\neg C(b), D(a)\}$. Thus, for the KB extensions $\Sigma_1 = \langle \{C \sqsubseteq D\}, \{C(a), D(b), C(b), D(a)\} \rangle$, and $\Sigma_2 = \langle \{C \sqsubseteq D\}, \{C(a), D(b), \neg C(b), D(a)\} \rangle$, their respective sets of EαKs will be $\Sigma_1 {\perp\!\!\!\perp} D(a) = \{\langle \{C \sqsubseteq D\}, \{C(a)\} \rangle, \langle \{\}, \{D(a)\} \rangle\} = \Sigma_2 {\perp\!\!\!\perp} D(a)$.*

Note that there is just one α-minimal extension $\Sigma_{\mathcal{I}^\alpha} = \langle \{C \sqsubseteq D\}, \{C(a), D(b), D(a)\} \rangle$. Hence, its related set of EαKs $\Sigma_{\mathcal{I}^\alpha} {\perp\!\!\!\perp} D(a)$ coincides with $\Sigma_1 {\perp\!\!\!\perp} D(a)$ and $\Sigma_2 {\perp\!\!\!\perp} D(a)$.

Example 7 (Ex. 2 cont.) *Let $\mathcal{I}_1^\alpha, \mathcal{I}_2^\alpha \in \mathcal{M}(\Sigma)$ be the two α-minimal models with domain $\Delta^\mathcal{I}$, such that $b \in Re^{\mathcal{I}_1^\alpha}$ and $b \in \neg Re^{\mathcal{I}_2^\alpha}$. One E$\alpha$K appears in each α-minimal extension: $\mathcal{K}_1 = \langle \{\}, \{SU(b,c), \neg Re(c), Re(b)\} \rangle$, $\mathcal{K}_1 \subseteq \Sigma_{\mathcal{I}_1^\alpha}$.*

$\mathcal{K}_2 = \langle \{\}, \{Re(a), SU(a,b)\} \cup \{\neg Re(b)\} \rangle$, $\mathcal{K}_2 \subseteq \Sigma_{\mathcal{I}_2^\alpha}$. Finally, the related sets of EαKs are: $\Sigma_{\mathcal{I}_1^\alpha} {\perp\!\!\!\perp} \alpha = \{\mathcal{K}_1\}$, and $\Sigma_{\mathcal{I}_2^\alpha} {\perp\!\!\!\perp} \alpha = \{\mathcal{K}_2\}$.

In order to avoid α inferences, we need to analyze every EαK in some set $\Sigma_\mathcal{I} {\perp\!\!\!\perp} \alpha$. A function "$\gamma$", namely *selection function*, determines the appropriate model from where the KB extension and the set of EαKs are built. Such function should apply the restrictions in Def. 4 guided by some *model preference criterion*, namely "$\prec$", used to univocally determine the "most profitable" α-minimal model.

Definition 5 (Model Selection Function "γ") *Let Σ be a KB, α, a sentence, and "$\prec$", a model preference criterion. A function "γ" is a "model selection function" determined by "$\prec$" iff $\gamma^\alpha(\Sigma) = \mathcal{I}^\alpha \in \mathcal{M}(\Sigma)$, where $\mathcal{I}^\alpha$ is an α-minimal model and for no other model $\mathcal{I} \in \mathcal{M}(\Sigma)$, it follows $\mathcal{I} \prec \mathcal{I}^\alpha$. The selected model $\mathcal{I}^\alpha$ will be noted as $\mathcal{I}^\gamma$.*

As stated before, the intuition behind a model-based contraction is to impact the cardinality of the set of models satisfying the KB such that some new admitted model will fail to satisfy α in the resultant KB. For this matter, we will generate the set of EαKs $\Sigma_{\mathcal{I}^\gamma} {\perp\!\!\!\perp} \alpha$ of the KB extension obtained from the model $\mathcal{I}^\gamma$ selected by the function "γ". Afterwards, we will define a mapping "σ" from the set $\Sigma_{\mathcal{I}^\gamma} {\perp\!\!\!\perp} \alpha$ to a sub-KB to be further eliminated from the original KB.

In this sense, two levels of deletions are to be considered: (1) axioms, or (2) assertions supporting the inference of α from every EαK in the selected KB extension. The foundations of such kind of contractions have been explained before: axioms are considered outdated and assertions are chosen when no axiom is considered in the EαK.

Definition 6 (Model Incision Function "σ") *Let $\mathcal{L}$ be a DL, $\Sigma \subseteq \mathcal{L}_{\mathcal{T}} \times \mathcal{L}_{\mathcal{A}}$, a KB, $\alpha \in \mathcal{L}$, a sentence, $\mathcal{I}^{\gamma}$, the model selected by the model selection function "γ", and $\sigma : 2^{\mathcal{L}_{\mathcal{T}} \times \mathcal{L}_{\mathcal{A}}} \longrightarrow \mathcal{L}_{\mathcal{T}} \times \mathcal{L}_{\mathcal{A}}$, a function mapping from $(\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} \alpha)$ to a KB $\langle \mathcal{T}_{\sigma}, \mathcal{A}_{\sigma} \rangle$. Then, "$\sigma$" is a "model incision function" iff it verifies:*

(1) $\sigma(\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} \alpha) \subseteq (\bigcup_{\mathcal{K} \in (\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} \alpha)} \mathcal{K}) \cap \Sigma$.

(2) *For all EαK $\mathcal{K} \in (\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} \alpha)$ it follows:*

a) $\mathcal{T}_{\mathcal{K}} \subseteq \mathcal{T}_{\sigma}$, and

b) if $\mathcal{T}_{\mathcal{K}} = \emptyset$ and $\mathrm{F}_{\mathcal{K}} \neq \emptyset$ then $\mathrm{F}_{\mathcal{K}} \cap \mathcal{A}_{\sigma} \neq \emptyset$.

Note that, from Def. 6, the fact of considering assertions only when no axiom exists, may be used to specify the model preference criterion "$\prec$" from Def. 5. Therefore, an option to identify the "most profitable" α-minimal model may be to analyze which $\mathcal{I}^{\alpha}$ leads to the KB extension with less EαKs $\mathcal{K}$ verifying $\mathcal{T}_{\mathcal{K}} = \emptyset$. This would avoid instance data loss, passing more axioms to be debugged by the second change sub-operation.

Definition 7 (Model-Based Contraction) *Let $\mathcal{L}$ be a DL, $\Sigma \subseteq \mathcal{L}_{\mathcal{T}} \times \mathcal{L}_{\mathcal{A}}$, a KB, $\alpha \in \mathcal{L}$, a sentence, $\Sigma_{\mathcal{I}^{\gamma}}$, the KB extended through the selected model $\mathcal{I}^{\gamma}$; and "σ", a model incision function. The operator "$\ominus_{\sigma}$", referred as model-based contraction determined by "σ", is defined as $\Sigma \ominus_{\sigma} \alpha = \Sigma \backslash \sigma(\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} \alpha)$.*

Finally, "$\ominus$" is a "model-based contraction operator" for Σ iff there exists a model incision function "σ" such that $\Sigma \ominus \alpha = \Sigma \ominus_{\sigma} \alpha$ for all sentence α.

Example 8 (Ex. 6 cont.) *From Def. 5 we have only one α-minimal model $\mathcal{I}^{\alpha} = \mathcal{I}^{\gamma}$ determining $\Sigma_{\mathcal{I}^{\gamma}}$. Later on, there are two EαKs in $\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} \alpha$, from which one is not considered by the incision function given that its related sets $\mathcal{T}_{\mathcal{K}}$ and $\mathrm{F}_{\mathcal{K}}$ are empty. Finally, since the other EαK considers terminologies (axioms), from Def. 6 we have that $\sigma(\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} \alpha) = \langle \{C \sqsubseteq D\}, \{\} \rangle$. Finally, the resultant KB would be $\Sigma' = \Sigma \ominus D(a) = \Sigma \backslash \sigma(\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} D(a)) = \langle \{\}, \{C(a), D(b)\} \rangle$.*

Axiomatic Characterization

As the basis for the axiomatization, we extend the basic postulates for bases given in (Hansson 1999).

(Inclusion) $\Sigma \ominus \alpha \subseteq \Sigma$.

(Success) [5] If $\not\models \alpha$ then $\Sigma \ominus \alpha \not\models \alpha$.

(Core Retainment) [6] If $\beta \in \Sigma$ and $\beta \notin \Sigma \ominus \alpha$ then there is some $H \subseteq \Sigma_{\mathcal{I}^{\gamma}}$ such that $H \not\models \alpha$ but $(H \cup \{\beta\}) \models \alpha$.

(Uniformity) For every $H \subseteq \Sigma_{\mathcal{I}^{\gamma}}$ it is verified that if $H \models \alpha$ *iff* $H \models \beta$ then $\Sigma \ominus \alpha = \Sigma \ominus \beta$.

[5] We use $\models \alpha$ to denote α as tautological.

[6] When β is an axiom, $(H \cup \langle \{\beta\}, \emptyset \rangle) \models \alpha$ follows; whereas when β is an assertion, it follows $(H \cup \langle \emptyset, \{\beta\} \rangle) \models \alpha$.

As is shown below, a model-based contraction operator defined so far does not guarantee success.

Example 9 (Ex. 7 cont.) *Suppose $\mathcal{I}^{\gamma} = \mathcal{I}_1^{\alpha}$, thus $\neg Re(c)$ could be chosen by the incision "σ". In such a case, the resultant KB Σ' would admit new models $\mathcal{I}'$ where $c \in Re^{\mathcal{I}'}$. Hence a new EαK will appear: $H \cup \langle \emptyset, \{Re(c)\} \rangle$, where $H = \langle \emptyset, \{SU(c,d), \neg Re(d)\} \rangle$, $H \subseteq \Sigma'$, but also $H \subseteq \Sigma$.*

A contraction operation not guaranteing success was proposed in (Fermé & Hansson 2001), but in the context of ontology evolution, we believe that success should be a must.

After a model-based contraction deletes some beliefs, it may fail to guarantee success if some subset $H \subseteq \Sigma$ along with the negation of some beliefs chosen by "σ", turns out being a new α-proof in the resultant KB. These kind of subsets are referred as *shielding sets* of information. Note that a shielding set is also a KB.

For a shielding set H, it follows $H \cup \langle \emptyset, \widehat{\mathcal{A}_{\sigma}} \rangle \models \alpha$, if $\widehat{\mathcal{A}_{\sigma}}$ is a *disagreement set* of the assertions chosen by "σ". Intuitively, a disagreement set negates some of those assertions and keeps the rest of them as they are.

Definition 8 (Disagreement set) *Given an ABox $\mathcal{A}$, the set $\delta(\mathcal{A})$ of disagreement sets $\widehat{\mathcal{A}}$ is:*

$$\delta(\mathcal{A}) = \{\widehat{\mathcal{A}} \neq \mathcal{A} | \forall \beta, \beta \in (\mathcal{A} \backslash \widehat{\mathcal{A}}) \textit{ iff } \neg \beta \in (\widehat{\mathcal{A}} \backslash \mathcal{A})\}$$

Note that a disagreement set $\widehat{\mathcal{A}}$ may be also referred as a disagreement ABox. Now we can formally define the *shielding sets* by means of some disagreement set of $\mathcal{A}_{\Sigma}$.

Definition 9 (Shielding Set) *A set $H \subseteq \Sigma$ is a "shielding set" iff $H \cup \langle \emptyset, \widehat{\mathcal{A}} \rangle \models \alpha$, for some $\widehat{\mathcal{A}} \in \delta(\mathcal{A})$, $\mathcal{A} \subseteq \mathcal{A}_{\Sigma}$.*

From an incision, some of its disagreements may "activate" a shielding set from Σ. In such a situation, the resultant KB will keep the previous EαKs, and for the new triggered models, a new EαK will appear. Thus, by restricting incisions, we avoid any shielding set to be activated.

(Anti-Shielding) There is no $H \subseteq \Sigma \backslash \sigma(\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} \alpha)$ such that $H \cup \langle \emptyset, \widehat{\mathcal{A}_{\sigma}} \rangle \models \alpha$, for some $\widehat{\mathcal{A}_{\sigma}} \in \delta(\mathcal{A}_{\sigma})$.[7]

This property anticipates the generation of a new model satisfying α in the resultant KB. If this happens, *anti-shielding* restricts the incision function in order to avoid the validity of that model. Hence, by considering a model-based contraction determined by a model incision function which satisfies *anti-shielding*, we guarantee success.

Definition 10 (Anti-Shielding Model-Based Contraction) *Let "$\ominus$" be a model-based contraction operator determined by a model incision function "σ". The function "σ" guaranties anti-shielding iff "$\ominus$" is an "anti-shielding model-based contraction operator".*

In the following example it is shown how a model incision function is restricted by anti-shielding.

Example 10 (Ex. 9 cont.) *Assume $\mathcal{A} = \{\neg Re(c)\}$, hence $\delta(\mathcal{A}) = \{\{Re(c)\}\}$. Thus, given $H \cup \langle \emptyset, \widehat{\mathcal{A}} \rangle \models \alpha$, where*

[7] Note that $\sigma(\Sigma_{\mathcal{I}^{\gamma}} {\perp\!\!\!\perp} \alpha) = \langle \mathcal{T}_{\sigma}, \mathcal{A}_{\sigma} \rangle$.

$\widehat{\mathcal{A}} \in \delta(\mathcal{A})$, if $\mathcal{A}_\sigma = \mathcal{A}$, where $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \langle \mathcal{T}_\sigma, \mathcal{A}_\sigma \rangle$, it is clear that "$\sigma$" does not guarantee anti-shielding. Finally, $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \langle \{\}, \{SU(b,c)\} \rangle$. Hence, for the resultant KB $\Sigma \ominus \alpha = \langle \emptyset, \{SU(a,b), SU(c,d), Re(a), \neg Re(c), \neg Re(d)\} \rangle$ success is assured, i.e., $\Sigma \ominus \alpha \not\models \alpha$.

Proposition 2 *An operator "$\ominus$" is an anti-shielding model-based contraction iff it guaranties success.*

Proof: Given a KB $\Sigma = \langle \mathcal{T}_\Sigma, \mathcal{A}_\Sigma \rangle$, and a sentence α, we will assume $\Sigma \models \alpha$, and $\not\models \alpha$. Moreover, a new contracted KB $\Sigma' = \Sigma \ominus \alpha$ is such that $\Sigma' = \langle \mathcal{T}_{\Sigma'}, \mathcal{A}_{\Sigma'} \rangle$, where $\mathcal{T}_{\Sigma'}$ and $\mathcal{A}_{\Sigma'}$ are the contracted TBox and ABox, respectively.

($\Rightarrow$) For the first part, if "$\ominus$" is an anti-shielding model-based contraction then it guaranties success, let us assume to the contrary that "$\ominus$" does not guarantee success, *i.e.*, $\Sigma' \models \alpha$. It follows that every model $\mathcal{I}' \in \mathcal{M}(\Sigma')$ satisfies α. Thus, in every KB extension ${\Sigma'}_{\mathcal{I}'^\alpha}$ there exists at least one EαK $\mathcal{K} = \langle \mathcal{T}_\mathcal{K}, \mathrm{F}_\mathcal{K} \cup \mathrm{S}_\mathcal{K} \rangle$ such that $H = \langle \mathcal{T}_\mathcal{K}, \mathrm{F}_\mathcal{K} \rangle$, where $\mathcal{T}_\mathcal{K} \subseteq \mathcal{T}_{\Sigma'}$ and $\mathrm{F}_\mathcal{K} \subseteq \mathcal{A}_{\Sigma'}$, and a subset of assumed beliefs $\mathrm{S}_\mathcal{K} \subseteq \mathrm{S}_{\mathcal{I}'^\alpha}$, where $\mathcal{I}'^\alpha \in \mathcal{M}(\Sigma')$. Let $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \langle \mathcal{T}_\sigma, \mathcal{A}_\sigma \rangle$ be the KB determined by model incision function, such that $\mathcal{T}_\sigma \subseteq \mathcal{T}_\Sigma$ and $\mathcal{A}_\sigma \subseteq \mathcal{A}_\Sigma$.

By Def. 7, $\mathcal{A}_\sigma \not\subseteq \mathcal{T}_{\Sigma'}$, then some ${\Sigma'}_{\mathcal{I}'^\alpha}$ may consider any disagreement $\widehat{\mathcal{A}_\sigma} \in \delta(\mathcal{A}_\sigma)$, as well as $\mathcal{A}_\sigma$ itself. Therefore, $\mathrm{S}_\mathcal{K} \subseteq \widehat{\mathcal{A}_\sigma}$ holds for any $\widehat{\mathcal{A}_\sigma} \in \delta(\mathcal{A}_\sigma)$. Hence, $\mathcal{K} \models \alpha$, and in particular $H \cup \langle \emptyset, \widehat{\mathcal{A}_\sigma} \rangle \models \alpha$ hold, contradicting the anti-shielding postulate.

($\Leftarrow$) For the opposite way, if "$\ominus$" does guarantee success then it is an anti-shielding model-based contraction. We know that every $\mathcal{I} \in \mathcal{M}(\Sigma)$ satisfies α, but since $\Sigma' \not\models \alpha$, there is some $\mathcal{I}' \in \mathcal{M}(\Sigma')$ which does not satisfy α. Thus, it is plausible to assume that the interpretation $\mathcal{I}'$ ends up being a model for Σ' as a result of the contraction operation, *i.e.*, $\Sigma \backslash \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$ where $\mathcal{I}^\gamma \in \mathcal{M}(\Sigma)$, and $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \langle \mathcal{T}_\sigma, \mathcal{A}_\sigma \rangle$. Thereafter, for every EαK $\mathcal{K} \in (\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$ we have two standpoints from Def. 6: $a) \mathcal{T}_\mathcal{K} \subseteq \mathcal{T}_\sigma$, and $b)$ if $\mathcal{T}_\mathcal{K} = \emptyset$ and $\mathrm{F}_\mathcal{K} \neq \emptyset$ then $\mathrm{F}_\mathcal{K} \cap \mathcal{A}_\sigma \neq \emptyset$.

Since anti-shielding affects only assertional knowledge in $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$, the former case is verified trivially. From case $b)$, we have $\mathcal{A}_\sigma \subseteq \Sigma$ and $\mathcal{A}_\sigma \not\subseteq \Sigma'$. This means that $\mathcal{A}_\sigma$ is satisfied by every model $\mathcal{I} \in \mathcal{M}(\Sigma)$, but there are some $\mathcal{I}' \in \mathcal{M}(\Sigma')$ that do not satisfy it. Therefore, for each $\beta \in \mathcal{A}_\sigma$ there is some $\mathcal{I}'$ satisfying $\neg\beta$. Moreover, there is at least one model $\mathcal{I}'$ for each disagreement $\widehat{\mathcal{A}_\sigma} \in \delta(\mathcal{A}_\sigma)$. Finally, each $\mathcal{I}'$ determines a KB extension ${\Sigma'}_{\mathcal{I}'}$ that do not contain any EαK for α. This means that ${\Sigma'}_{\mathcal{I}'} \not\models \alpha$ and since $\widehat{\mathcal{A}_\sigma} \subseteq \mathrm{S}_{\mathcal{I}'} \subseteq {\Sigma'}_{\mathcal{I}'}$, it is clear that there is no $H \subseteq \Sigma'$, *i.e.*, $H \subseteq \Sigma \backslash \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$ such that $H \cup \langle \emptyset, \widehat{\mathcal{A}_\sigma} \rangle \models \alpha$, as stated by anti-shielding. □

Theorem 1 (Anti-Shielding Model-Based Contraction) *An operator "$\ominus$" is an anti-shielding model-based contraction operator iff it satisfies success, inclusion, core retainment, and uniformity.*

Proof: **Construction-to-postulates:** Let "$\ominus$" be an anti-shielding model-based contraction for $\Sigma = \langle \mathcal{T}_\Sigma, \mathcal{A}_\Sigma \rangle$. We will show that it satisfies the four conditions of the theorem.

Success follows from the first part of Prop. 2, and inclusion, trivially from Def. 7. For core-retainment, suppose that $\beta \in \Sigma$ and $\beta \notin \Sigma \ominus \alpha$. Then it holds that $\beta \in \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$, given that $\Sigma \ominus \alpha = \Sigma \backslash \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$ by Def. 7. Therefore, by Def. 6, $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) \subseteq (\bigcup_{\mathcal{K} \in (\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)} \mathcal{K}) \cap \Sigma$, where $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \langle \mathcal{T}_\sigma, \mathcal{A}_\sigma \rangle$. Thus, there is some E$\alpha$K $\mathcal{K} \subseteq \Sigma_{\mathcal{I}^\gamma}$ such that $\beta \in \mathcal{K}$. It follows that $\beta \in \mathcal{T}_\Sigma$ or $\beta \in \mathcal{A}_\Sigma$. For the former, when $\beta \in \mathcal{T}_\Sigma$, we know $\beta \in \mathcal{T}_\mathcal{K}$ and following condition $(2a)$ in Def. 6, $\mathcal{T}_\mathcal{K} \subseteq \mathcal{T}_\sigma$ holds for every $\mathcal{K}$. For the latter, when $\beta \in \mathcal{A}_\Sigma$, we know $\beta \in \mathrm{F}_\mathcal{K}$ and following condition $(2b)$ in Def. 6, we know that $\mathcal{T}_\mathcal{K} = \emptyset$, $\mathrm{F}_\mathcal{K} \neq \emptyset$, and $\beta \in (\mathrm{F}_\mathcal{K} \cap \mathcal{A}_\sigma)$. Thus, in any case, it is proved the existence of some EαK $\mathcal{K} \subseteq \Sigma_{\mathcal{I}^\gamma}$ such that $\beta \in \mathcal{K}$. Let now consider $H = \mathcal{K} \backslash \{\beta\}$. It is clear that $H \subseteq \Sigma_{\mathcal{I}^\gamma}$, then $H \not\models \alpha$ but $H \cup \{\beta\} \models \alpha$ (from condition (3) in Def. 2) shows core-retainment is satisfied.

For uniformity, suppose that it holds for all subsets $B \subseteq \Sigma_{\mathcal{I}^\gamma}$ that $B \models \alpha$ if and only if $B \models \beta$. By Prop. 3, $\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha = \Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \beta$. It follows from this that $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \beta)$, and by the definition of "$\ominus$" that $\Sigma \ominus \alpha = \Sigma \ominus \beta$, so that uniformity is satisfied.

Postulates-to-construction: Let "$\ominus$" and Σ be such that the four conditions of the theorem are satisfied. We are going to show that "$\ominus$" is an anti-shielding model-based contraction. For that purpose, let σ be such that for all α:

$$\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \Sigma \backslash (\Sigma \ominus \alpha).$$

This follows from inclusion $(\Sigma \ominus \alpha \subseteq \Sigma)$ and $\Sigma \ominus \alpha = \Sigma \backslash \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$ as posed by Def. 7. Thereafter, we need to verify that σ is an anti-shielding model incision function for Σ. To be that, both "γ" and "σ" must be functions, and "σ" also satisfy (1), and (2) from Def. 6, and anti-shielding.

Proof that "γ" and "σ" are functions: Let α and β be two sentences such that $\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha = \Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \beta$. We need to show that $\gamma^\alpha = \gamma^\beta = \mathcal{I}^\gamma$ and $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \beta)$. It follows from $\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha = \Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \beta$, by Prop. 3, that every subset $B \subseteq \Sigma_{\mathcal{I}^\gamma}$ implies α if and only if it implies β. Thus, by uniformity, $\Sigma \ominus \alpha = \Sigma \ominus \beta$. Hence, from the definition of "γ", "σ", and "$\ominus$" it follows that $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \beta)$.

Proof that (1) is satisfied: We will show that $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) \subseteq (\bigcup_{\mathcal{K} \in (\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)} \mathcal{K}) \cap \Sigma$. Let $\beta \in \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$, it follows from core-retainment (given its preconditions $\beta \in \Sigma$ and $\beta \notin \Sigma \ominus \alpha$) that there is some $H \subseteq \Sigma_{\mathcal{I}^\gamma}$ such that $H \not\models \alpha$ but $H \cup \{\beta\} \models \alpha$. Hence, it follows that there is some EαK $\mathcal{K}$ such that $\beta \in \mathcal{K} \in (\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$. Thus, $\beta \in \bigcup_{\mathcal{K} \in (\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)} \mathcal{K}$ and since $\beta \in \Sigma$, we conclude $\beta \in ((\bigcup_{\mathcal{K} \in (\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)} \mathcal{K}) \cap \Sigma)$.

Proof that (2) is satisfied: Suppose that $\langle \emptyset, \emptyset \rangle \neq \mathcal{K} \in \Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha$. It follows from this that $\not\models \alpha$. By success, $\Sigma \ominus \alpha \not\models \alpha$. Since $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) \subseteq \Sigma$ (where $\sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha) = \langle \mathcal{T}_\sigma, \mathcal{A}_\sigma \rangle$) and $\mathcal{K} \models \alpha$ we may conclude that $(\mathcal{T}_\mathcal{K} \cup \mathrm{F}_\mathcal{K}) \not\subseteq \Sigma \ominus \alpha$, *i.e.*, that there is some $\beta \in (\mathcal{T}_\mathcal{K} \cup \mathrm{F}_\mathcal{K})$ such that $\beta \notin \Sigma \ominus \alpha$. This leave us two options: $\beta \in \mathcal{T}_\sigma$ or $\beta \in \mathcal{A}_\sigma$. Besides, since $(\mathcal{T}_\mathcal{K} \cup \mathrm{F}_\mathcal{K}) \subseteq \Sigma$ it follows $\beta \in (\Sigma \backslash \Sigma \ominus \alpha)$, *i.e.*, $\beta \in \sigma(\Sigma_{\mathcal{I}^\gamma} \perp\!\!\!\perp \alpha)$. Therefore, we have also that $\beta \in \mathcal{T}_\mathcal{K}$ or $\beta \in \mathrm{F}_\mathcal{K}$. Hence, if $\beta \in \mathcal{T}_\mathcal{K}$, this is enough to prove $a)$. To the contrary, if $\beta \in \mathrm{F}_\mathcal{K}$, we have that $\mathcal{T}_\mathcal{K} = \emptyset$, $\mathrm{F}_\mathcal{K} \neq \emptyset$, and $\beta \in (\mathrm{F}_\mathcal{K} \cap \mathcal{A}_\sigma)$. Finally $(\mathrm{F}_\mathcal{K} \cap \mathcal{A}_\sigma) \neq \emptyset$.

Proof that "σ" guaranties anti-shielding: Given success, it follows directly from the second part of Prop. 2. $\Box$

The following equivalence is similar to that introduced in (Hansson 1999). Its proof is absence due to space reasons.

Proposition 3 *The following conditions are equivalent:*

(1) $\Sigma_{\mathcal{I}^\gamma}{}^{\perp\!\!\perp}\alpha$=$\Sigma_{\mathcal{I}^\gamma}{}^{\perp\!\!\perp}\beta$

(2) *For all subsets $H \subseteq \Sigma_{\mathcal{I}^\gamma}$: $H \models \alpha$ iff $H \models \beta$.*

The following theorems are related to the principles proposed in (Dalal 1988). Theorem 2 assures that any resultant KB from a model-based contraction operation will conform the DL used for the previous KB, whereas Theorem 3 abstract away from the DL used, stating that any pair of logically equivalent knowledge will be equally treated.

Theorem 2 (Adequacy of Representation) *Let $\mathcal{L}$ be an $\mathcal{AL}[\mathcal{U}][\mathcal{E}][\mathcal{C}]$ DL. For any KB $\Sigma \subseteq \mathcal{L}_\mathcal{T} \times \mathcal{L}_\mathcal{A}$ and any sentence $\alpha \in \mathcal{L}$, it follows $(\Sigma \ominus \alpha) \subseteq \mathcal{L} \times \mathcal{L}$.*

Theorem 3 (Irrelevance of Syntax) *Let $\mathcal{L}, \mathcal{L}'$ be two $\mathcal{AL}[\mathcal{U}][\mathcal{E}][\mathcal{C}]$ DLs, $\Sigma \subseteq \mathcal{L}_\mathcal{T} \times \mathcal{L}_\mathcal{A}$ and $\Sigma' \subseteq \mathcal{L}_\mathcal{T}{}' \times \mathcal{L}_\mathcal{A}{}'$, two KBs, and $\alpha \in \mathcal{L}$ and $\alpha' \in \mathcal{L}'$, two such DL sentences. If $\alpha \approx \alpha'$ (where $\approx$ means logically equivalent to) and for every $\beta \in \Sigma$ and every $\beta' \in \Sigma'$, $\beta \approx \beta'$, then for every $\beta_R \in (\Sigma \ominus \alpha)$ and every $\beta'_R \in (\Sigma' \ominus \alpha')$, $\beta_R \approx \beta'_R$.*

Algorithm Specification

Since our theory relies on the proper selection of an α-minimal model $\mathcal{I}^\gamma \in \mathcal{M}(\Sigma)$, it is important to propose an algorithm implementing the selection function. An interesting approach would be to take advantage of the tableau algorithm used by the DL's subjacent inference engine, in such a way that $\mathcal{I}^\gamma$ turns out being its outcome at the time the satisfiability of α is being checked. This would provide an important shortcut in favor of the computability, such that finding the proper model $\mathcal{I}^\gamma$ would be attached to the time of computing the satisfiability checking of the sentence α.

Notice that the (canonical) model identified by a classical tableau procedure has the form of a model $\mathcal{I}^\alpha$, thus we will trivially assume the devised model as the one selected by the model selection function "γ", such that $\mathcal{I}^\alpha = \mathcal{I}^\gamma$. Hence, its related α-minimal KB extension $\Sigma_{\mathcal{I}^\gamma}$ may be generated and therefore it could be checked which of the EαKs obtained by the tableau process is included in $\Sigma_{\mathcal{I}^\gamma}$, thus generating the set of EαKs $\Sigma_{\mathcal{I}^\gamma}{}^{\perp\!\!\perp}\alpha$. Hence, a model incision function "σ" may be applied obtaining $\sigma(\Sigma_{\mathcal{I}^\gamma}{}^{\perp\!\!\perp}\alpha)$ and finally the contraction $\Sigma \ominus \alpha$ is resolved.

A special mention should be done regarding the anti-shielding property. As seen before, the anti-shielding validation is directly related to the assertional knowledge the incision function chooses to eliminate, and the sentence α to be contracted . The main problem appears when the incised assertions $\mathcal{A}_\sigma$, and its disagreement sets $\widehat{\mathcal{A}_\sigma}$, may conform new models satisfying α. That is, if the sentence α to be contracted considers a concept C and also some role R whose range is in its complement $\neg C$ (see Ex. 2), this may provoke the disagreement sets to be part of the new α-minimal extensions of the resultant KB.

To solve this situation, a simple heuristic may consider to avoid any incision of a concept if its complement is also considered in the satisfiability checking of the tableau machinery, disregarding both are relating different individual names. This is specified below.

Anti-Shielding Rule
Condition: $\mathcal{A}$ contains $A(x)$, and $\forall R.\neg A(y)$ or $\exists R.\neg A(y)$, for any individual names x, y such that $x \neq y$.
Action: $\mathcal{A}' \leftarrow \mathcal{A} \backslash \{A(x), \neg A(y)\}$.

Notice that the anti-shielding rule will be used after obtaining a closed constraint system, and it will always leave a non-empty ABox $\mathcal{A}'$ since such a rule could only be applied in the presence of some role $R(x, y)$. Therefore, the model incision function would have at least $R(x, y)$ to choose.

Algorithm 1 Calculate $\Sigma' = \Sigma \ominus \alpha$.
Input: Σ, α.
Output: Σ'.
$\overrightarrow{S} \leftarrow \mathit{tableauProc}(\Sigma, \alpha)$.
if $\overrightarrow{S}$ is closed **then**
$\mathcal{I}^\alpha \leftarrow \mathit{canonicalModel}(\Sigma, \overrightarrow{S})$.
$\sigma(\Sigma_{\mathcal{I}^\gamma}{}^{\perp\!\!\perp}\alpha) \leftarrow \mathit{antiShIncision}(\Sigma, \mathcal{I}^\alpha, \overrightarrow{S})$.
$\Sigma' \leftarrow \Sigma \backslash \sigma(\Sigma_{\mathcal{I}^\gamma}{}^{\perp\!\!\perp}\alpha)$.
else
$\Sigma' \leftarrow \Sigma$.
end if

The $\mathit{antiShIncision}$ procedure identifies each EαK from the closed constraint system conforming the model $\mathcal{I}^\alpha$ and the KB Σ. This is done by recognizing each EαK at a time, in order to maintain the same space requirement of the related tableau procedure. After one EαK is identified, it is viewed as an instantiation from the closed constraint system, then the Anti-Shielding Rule above is applied to the EαK restricting the domain of the incision wrt. assertions. Consequently, the model incision is applied to the remainder of the EαK at issue, in accordance to Def. 6. Afterwards, the closed constraint system (which remains intact) will determine the next recognition of a new EαK.

Theorem 4 (Model-Based Contractions Complexity) *Let $\mathcal{L}$ be an $\mathcal{ALC}$ DL, $\Sigma \subseteq \mathcal{L}$, a KB, $\alpha \in \mathcal{L}$, a sentence, and "$\ominus$", an anti-shielding model-based contraction operator. The complexity of $\Sigma \ominus \alpha$ is PSPACE-complete.*

Proof sketch: $\mathcal{ALC}$ DLs have been proved to be PSPACE-complete for satisfiability of concepts descriptions, following the related tableau algorithm. Thus, since Algorithm 1 calls once to the tableau procedure, we should analyze that the rest of the algorithm could be computed in polynomial space as being required by the procedure $\mathit{tableauProc}$.

The most problematic procedure in our algorithm may be $\mathit{antiShIncision}$. Conforming the given canonical model and the resultant constraint system, $\mathit{antiShIncision}$ chooses the related knowledge to each EαK by calculating, and considering them, each at a time (this is done with no need to calculate the related KB extension). Afterwards, since each EαK fits the space of the constraint system considered, it follows that $antiShIncision$ is executed in the polynomial space required before. Finally, Alg.1 for $\mathcal{ALC}$ DLs is proved to be PSPACE-complete. □

Related Work

Ontology revision is currently an interesting topic in which belief revision meets description logics. In the last few years, several articles in this area has been published. For instance, in (Flouris *et al.* 2006), incoherence and inconsistency of ontologies are formally presented. Based on the distinction between coherent and consistent negation, a set of postulates for revising DLs is proposed, although no operator is specified. Recently in (Qi *et al.* 2008), a kernel revision operator for terminologies was presented, there an incision function is specified to delete axioms avoiding a terminology to evolve incoherently. (Ribeiro & Wassermann 2007) presents a similar approach in which an incision is performed over terminologies dealing with inconsistency. Similarly, in (Haase *et al.* 2005), based on a selection function different sub-ontologies are identified to consistently incorporate a given new axiom.

In general, most of the ontology change operators proposed so far are based on the notions of MIPS and MUPS. Such constructions were originally presented in (Schlobach & Cornet 2003) as a debug tool for pinpointing terminological errors to correct inconsistencies. In contrast, in (Meyer, Lee, & Booth 2005) the KB is weakened and inconsistencies are also tolerated.

In our approach, we manage the evolution in ontologies by considering not only axioms but also assertional knowledge. Moreover, a model-based contraction addresses in advance inconsistencies and incoherencies, that is, statements eliminated from the ontology avoid as much as possible instance data loss (following minimal change). Afterwards, the intention of the complete change operator is to accommodate the deleted axioms to avoid incoherency, and further reincorporate them. This latter sub-task of the change operator is part of our ongoing work in the matter.

Similar to the construction of α-kernels, MUPS are defined as minimal inconsistent sets for a given atomic concept. This structure is constructed with the aim of restoring satisfiability to an unsatisfiable concept definition. In our approach, we anticipate the change and prepare the ontology to accept the new information consistently, leaving the accommodation of outdated axioms to the completion of the ontology change operation. This is the purpose of model-based contractions, defined as a functional part of the general change operation that makes effective an ontological change.

As stated before, model-based contractions are motivated by kernel contractions in their intuitions of breaking minimal proofs for a given sentence, but despite both constructions delete beliefs from minimal proofs, they are semantically different. In kernel contractions, deletions break or cut proofs. However, in model-based contractions deletions imply a variation of the set of models and therefore the generation of new associated KB extensions, while the original proofs remain.

In (Dalal 1988) a KB revision operation was semantically specified at the knowledge level by considering model-theoretic semantics. Although, as stated by the author, no consideration about differential treatment of certain atoms, and even formulae, is taken into account, so that some beliefs could be more easily given up.

The six AGM basic postulates for contractions (Alchourrón, Gärdenfors, & Makinson 1985) were supposed to capture the intuition behind any contraction operation on a belief set (closed under logical consequence). But recently in (Flouris February 2006), it was shown that some Tarskian logics –non-AGM Compliant logics– do not admit a contraction operation satisfying the six AGM postulates. However, they admit contractions without recovery.

Regarding DLs, we are particularly interested in very expressive logics like $\mathcal{SHIF}(\mathcal{D})$ and $\mathcal{SHOIN}(\mathcal{D})$, which are shown to be equivalent to OWL-Lite and OWL-DL (Horrocks & Patel-Schneider 2003), the two OWL sub-languages for which complete reasoners are known. Such DLs are known to be non-AGM Compliant (Flouris February 2006), but since this requires to guarantee recovery, the alternative to find some replacement postulate appears sensible.

In particular, recovery has been the most problematic postulate, standing unnaturally for the principle of minimal change. For instance, in (Hansson 1999) regarding bases, core-retainment have been proposed as a substitute for recovery, while uniformity stands for extensionality.

Discussion

In contrast to item $2a)$ in Def. 6, it seems enough for it to be defined as: "If $\mathcal{T}_{\mathcal{K}} \neq \emptyset$ then $\mathcal{T}_{\mathcal{K}} \cap \mathcal{T}_{\sigma} \neq \emptyset$, and... ".

Although that would (apparently) follow minimal change, such decision would be a detriment to minimal change wrt. the complete ontology change operator, as will be seen below. In consequence, we decided to take complete terminologies as stated by the following intuition: when an axiom ends up being unsatisfiable, a contradiction appears while checking its unfolded version. Since that terminology is part of a minimal α-proof, each of the axioms considered –along with the sentence to be incorporated– are interrelated, as inferential steps to infer a minimal incoherence.

Axiom unfolding may provoke an explosion in the size of the search space leading to a notable degradation in performance. For that reason, a careful analysis is required to provide an efficient methodology. In (Tsarkov, Horrocks, & Patel-Schneider 2007), a complete overview of lazy unfolding and other reasoning optimizations are described.

By analyzing a unique unfolded axiom it let us to identify the exact point in which the axiom turns to unsatisfiability. That is, the terminology from each EαK along with the sentence φ is a minimal incoherence preserving sub-terminology (MIPS). After that, technics from ontology debugging could be applied to restore coherence to the sub-terminology. Finally, it is incorporated to the KB.

For instance, in Ex. 1, we have axiom $D_3 \sqsubseteq C \sqcap \neg D_2$ in an EαK. Considering the sentence $\varphi = C \sqsubseteq D_2$, note that $\{D_3 \sqsubseteq C \sqcap \neg D_2, C \sqsubseteq D_2\}$ is a MIPS. It is clear that the unfolded axiom $D_3 \sqsubseteq D_2 \sqcap \neg D_2$ is unsatisfiable. Later on, coherency may be restored by assuming the unfolded axiom as $D_3 \sqsubseteq D_2$, which means that the repaired sub-terminology ends up being $\{D_3 \sqsubseteq C, C \sqsubseteq D_2\}$. Finally, the repaired terminology may be reincorporated to the evolved KB, which would end up consistent and coherent.

Note that, this (apparently) drawback on minimal change wrt. the contraction does not contradict *core-retainment*. The matter discussed above is part of the ongoing work, and is proposed as future work for the completion of the ontology change operation.

Conclusions and Future Work

In this paper we have proposed a new contraction operator of model-theoretic semantics, dedicated to avoid the inference of a sentence α in a specific ontology, both expressed in some description language as $\mathcal{AL}[\mathcal{U}][\mathcal{E}][\mathcal{C}]$.

The process modeled by model-based contractions may be summed up as follows: from an ontology $\mathcal{O}$ and some finite model $\mathcal{I}$, we extend the ontology to $\mathcal{O}_{\mathcal{I}}$. In it, different sub-ontologies, namely EαKs $\mathcal{K}_1, \mathcal{K}_2, \dots$ are identified. Analyzing the information in such sub-ontologies, an incision function "σ" determines a sub-ontology $\mathcal{O}_\sigma \subseteq \mathcal{O}$, such that $\mathcal{O} \models \alpha$, but $(\mathcal{O} \backslash \mathcal{O}_\sigma) \not\models \alpha$.

Our intention regarding the formal theory here provided, is to apply it on other more expressive DLs like $\mathcal{SHIF}(\mathcal{D})$ and $\mathcal{SHOIN}(\mathcal{D})$. This work is introductory in that sense. As part of its preliminary results, an algorithm was provided as a possible option towards its further realization. Such algorithm has been defined on top of the tableau procedure used by the related DL reasoner to find the appropriate model for the theory to be applied. Moreover, since the theory mostly relies on such a selection, the complexity results in terms of the space required to compute, depends on the tableaux algorithm used to reason.

In this sense, the model selection (and its related preference criterion "$\prec$") has been abstracted away from the algorithm, and assumed to be the canonical model recognized from the tableau procedure. For this matter, it becomes interesting to investigate the model preference criterion "$\prec$" to be specified. By determining such a criterion, the tableau procedure might be directed by specifying an order of the transformation rules to apply, and different properties to determine which constraint systems should be attended first.

Since ontologies are highly reusable distributed, generation of intermediate inconsistent states may be critical for any change operation. This is a considerable advantage that model-based contractions provide, since they "foretell" any undesired effect from the change operation, repairing it in advance. In this sense, adjustment of outdated axioms is part of our future work in the field of ontology debugging.

This proposal would be incomplete, without considering model-based contractions as a sub-operation of a broader ontology change operator. Indeed, some assumptions made relying on that consideration, are sensible only for that matter. Future work also involves the formalization of the ontology change operation, along with a set of general principles for the evolution, to state a set of postulates by which the new change operator may be axiomatically characterized.

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